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DELEGATION AND LEARNING

Bikramaditya Datta
(Indian Institute of Technology, Kanpur)

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Delegation and Learning*

Bikramaditya Datta†

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Abstract

A principal contracts with an agent to complete a task. The agent's ability to complete the task is uncertain and is learnt from the agent's performance in projects that the principal finances. Success however also depends on the quality of the project at hand, and quality is privately observed by the agent who is biased towards implementation. We characterize the optimal sequence of rewards in a relationship that tolerates an endogenously determined finite number of failures and incentivizes the agent to implement only good projects by specifying rewards for success as a function of past failures. The fact that success becomes less likely over time suggests that rewards for success should increase with past failures. However, this means that the agent can earn a rent by deviating and implementing a bad project, which is sure to fail. We show that this rent decreases with past failures and implies that optimal rewards are front-loaded. The optimal contract resembles the arrangements used in venture capital, where entrepreneurs must give up equity share in exchange for further funding following failure.

JEL Classification: D21, D82, D83, G31, G32

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†Corresponding author: Department of Economic Sciences, Indian Institute of Technology, Kanpur 208016, India. Email: bikramd@iitk.ac.in

1 Introduction

Consider a firm that evaluates entering a new business. The firm puts a manager in charge and finances projects related to this business, such as designing prototypes or testing specific markets. The manager is better informed about the quality of the projects - that is, their chances of succeeding - but there is uncertainty about the manager's fit to lead the firm's operations in the new business, and therefore about his ability to make projects succeed. In this paper, we study the optimal mechanism by which the firm delegates experimentation to the manager while learning about his ability.

The literature on contracting for experimentation has mostly focused on how to incentivize effort. Bergemann and Hege (1998, 2005) and Hörner and Samuelson (2013) for instance, study dynamic moral hazard models in which the principal finances the agent to work on projects but the agent can choose to divert cash for private benefits or equivalently not exert effort. However effort is only part of the overall incentive problem. In a managerial context, it is often likely the case that managers are industrious but the primary issue is determining how effective managers are in their tasks[1]. Our goal in this article is to understand how the firm can optimally incentivize the manager to implement the right projects while learning about the manager's ability. We analyze a situation where the manager is better informed about the quality of projects but biased relative to the firm. The experimentation component arises because both the firm and the manager learn about the manager's ability as the manager implements projects and they observe the projects' outcomes. The agency problem is related to the fact that the manager usually has better information about the quality of projects in which he can invest, but has incentives different from that of the firm. The firm would like the manager to wait for good projects and only take those up.

[1]See for instance Kaplan (1984), who considers effort-based models as inadequate for capturing incentive issues in management. Further PwC (2017) suggests that problem solving, creativity and innovation are among some of the most important skills as rated by CEOs across countries.

The manager on the other hand, benefits from working on projects regardless of quality. However good projects are not always available and hence the firm has to provide incentives for the manager to wait for the good projects. Further, one of the advantages of failure in projects is that the manager may earn further rents from future projects, while a success reveals the business is profitable for the firm and might lead the firm to place a specialist in charge of the business. Thus, the manager might want to take up projects which fail in order to postpone the completion of the learning phase. The problem of the firm is to find the optimal amount of funding and reward structure in order to incentivize the manager to select the right projects.

To study these issues, we develop a model in which a principal contracts with an agent to complete a task. The agent's ability to complete the task is unknown to both the principal and the agent. Completing the task requires success in a project. The agent's performance in a project depends both on his ability and the quality of the project at hand. In particular, only high ability agents have a chance of success in good quality projects, which arrive[2] stochastically and may not be available at any given point. Bad quality projects, which fail regardless of the ability of the agent, are always available. The quality of the projects available is privately observed by the agent before deciding which project to implement in any particular period. The principal only gets to observe whether a project implemented resulted in a success or a failure and not the quality of the project - this is the source of asymmetric information in the model.

Since only good quality projects can succeed, the principal would want the agent to only implement these projects. However, the agent is biased towards implementing projects regardless of quality, since he gets a private benefit regardless of quality of the project and his ability. In order to incentivize the agent to wait for a good project to arrive, the principal

[2]The arrival rate of good projects is independent of the agent's ability. Thus ability here refers to the agent's capability of succeeding in good projects.

offers reward for success in a project.

Failure in a project leads to a reduction in belief regarding the agent's ability and hence reduces the belief regarding probability of success in a project. This suggests that the rewards for success, needed to incentivize the agent to wait, should increase with past failures. However, this in turn creates an incentive for the agent to deviate and earn a rent. Suppose the principal expects the agent to implement only good projects. If the agent deviates and implements a bad project, then the resulting failure leads to a reduction in the principal's belief regarding the agent's ability, while the agent's belief about his ability remains unchanged[3]. Thus, the agent can ensure himself a strictly positive rent by this deviation.

The optimal contract has rewards for success decreasing with the number of past failures. Since success in a project completes the task and obviates the need for further project implementation, the agent will select a good project only if the rewards for succeeding in the project compensates him for the potential loss of continuation rents that selecting a good project makes more likely. These continuation rents not only include the private benefit from implementing projects but also rents due to possible divergence in beliefs described above. These factors combine to produce rewards for success which decrease with the number of past failures.

Another feature of the optimal contract is that, increasing the number of trials results in higher rewards to be paid to the agent for success. This is because increasing the number of trials implies that the potential loss of continuation rents from selecting a good project is higher for the agent. The loss in continuation rents is higher due to the possibility of getting private benefits from implementing a larger number of projects as well as earning higher rents due to the possibility of greater divergence of beliefs.

The optimal number of trials is determined by considering the trade-off between higher rent paid to the agent and better information obtained through increasing number of trials.

[3]This is because the agent knows that performance in bad projects is not indicative of ability.

Increasing the number of trials provides more opportunities for a high ability agent to succeed and thus reduces the probability that the agent was high ability but failed due to a lack of sufficient opportunities. However as discussed above, increasing the number of trials leads to higher bonuses paid to the agents for success. We further find that the optimal number of trials is an increasing function of the prior belief regarding the agent's ability and the payoff that the principal gets from success and is a decreasing function of the cost of implementing projects.

The model can also be used to analyze financial contracting between entrepreneurs and investors. An entrepreneur often has a better understanding of the products he can launch, but may receive private benefits, monetary or reputational, from launching products even when these are not profitable. Furthermore, it is initially unknown whether the entrepreneur has the necessary skills to make a good product succeed. Also, success by a entrepreneur often leads to his replacement[4] (Wasserman 2008). Thus we can apply the model to highlight some of the agency problems present in the relationship between the entrepreneur and the investor and illustrate how they impact the financial arrangements between them.

Empirical evidence on venture capital financing is consistent with the results obtained in the model. For instance, Kaplan and Strömberg (2003) find evidence that founders' cash flow rights decline over financing rounds and decrease as the firm performance worsens. This is consistent with the model's prediction that the rewards for the agent are a decreasing function of past failures. Similarly the result that a higher prior about the agent's ability leads to increased funding is consistent with the findings in the empirical literature on venture capital financing which suggest that entrepreneurs who have succeeded in the past are likely to get better deals. (Gompers, Kovner, Lerner and Scharfstein 2010).

Our paper contributes to the literature on contracting for experimentation. A finding that

[4]For instance, the first major task in a new venture is the development of its product or service. However, once the product is ready, the business often faces different challenges - marketing, sales and customer services and hence investors might want to put a different CEO in charge.

emerges from several papers in the experimentation literature focussed on incentivizing effort, is that optimal schemes for experimenting are lenient about failure and rewards for success should be a non-decreasing function of the number of past failures. Halac, Kartik and Liu (2016), for instance, study long-term contracts for experimentation, with adverse selection about the agent's ability and moral hazard about his effort choice. They find that the optimal bonus structure is either constant or back-loaded, that is the agent is rewarded more for later success. In contrast, we find that bonus structure should instead be front-loaded, that is the agent should be rewarded more for success after a fewer number of failures. The difference is driven by the fact that in our setting the agent gets a private benefit from implementing projects and hence must be compensated for the loss in continuation payoffs. Manso (2011) derives an optimal contract where the agent chooses between shirking, exploiting a well-known approach, or exploring a new approach. He finds that the optimal contract which induces the agent to try the new approach exhibits tolerance for early failure and rewards for long-term success. In contrast, in our setting the agent faces a choice between implementing a bad project or waiting for a good project to arrive to implement it. Our model suggests that tolerating early failures and rewarding long-term success might lead to adverse incentives for an agent who derives benefits from continuing to work on projects. In particular, our model brings into focus the incentive cost of giving an agent a higher number of opportunities to succeed.

The article is connected with the literature on delegation originating from Holmström (1977, 1984). An important focus in this literature has been on how to incentivize an biased agent with superior information to act in the principal's interest. We highlight the fact that delegation also allows us to learn about the agent's ability. Recently, there has been quite a few articles related to dynamic delegation - Hörner and Guo (2015), Lipnowski and Ramos (2015), Li, Matouschek & Powell (2017) - however these are in a repeated game setting and there is no learning component. An exception is Guo (2016). In her setting, the agent receives private information only once at the beginning of the game while in our setup the agent receives private information multiple times over the course of the game.

This article is also related to the literature on assessing managerial ability originating from Holmström (1999). The literature highlights that firms draw inferences about the manager's ability based on public signals. This in turn provides an incentive for the manager to take actions to distort the public signals. However typically the managers take actions which try and make them appear better than they are (or at least no worse than what they are)[5]. In contrast, in our model, managers benefit from the possibility of making the principal more pessimistic about his ability. In that respect, this is closer to the literature on belief manipulation[6] . The literature on belief manipulation has mostly focused on situations in which agents have to apply (hidden) effort. In contrast, our paper suggests another source - selecting bad projects - through which the agent might create a divergence between public belief about his ability and his own private belief and earn a rent on the basis of that.

The rest of the article is organized as follows. In section 2, we describe the model setup and solve a benchmark case with complete information. In section 3, we illustrate the basic insights and tradeoffs by considering the optimal contract which allows for one and two trials. In section 4, we derive the optimal contract for the general problem. In section 5, we present comparative statics results. Section 6 discusses some extensions and empirical implications and we conclude in section 7.

2 The Model

In this section, I describe the model setup and solve a benchmark case with complete information.

[5] See for example Hermalin (1993), Holmström and Ricart i Costa (1986).

[6] See for example Bergemann and Hege (2005), Bhaskar (2012), Wolf (2017).

2.1 Setup

There are two risk-neutral players: a (male) agent and a (female) principal. Both have a common discount factor $\delta \in (0,1)$. Time $t = 0, 1, 2...$ is discrete with an infinite horizon.

Ability*:* The ability of the agent is persistent and is either high or low. Neither the principal nor the agent knows the true ability - the initial common prior is that the agent is high ability with probability $\alpha_0 \in (0,1)$[7]. The agent's ability can be assessed through performance in projects.

Projects*:* Each period there are up to two types of projects available - "bad" and "good". A "bad" project fails regardless of the ability of the agent, whereas "good" project succeeds with probability $\gamma \in (0,1)$ if the agent is of high ability and fails otherwise. In each period, there is always a bad project available, whereas a good project is available with probability $p \in (0,1)$. The availability of projects is independent of the agent's ability[8]. The agent can implement up to one project each period. If a good project becomes available in a specific period and the agent chooses not to implement it that period, then the agent cannot implement that particular project in future periods either.

Payoffs*:* Following Zwiebel (1996), the agent gets a private benefit $b > 0$ per project implemented.[9] It costs the principal $c > 0$ to implement a project. Outside options per period for

[7]There are a few justifications for the common prior assumption. First, the agent's assessment of ability is based on past performance and hence is likely to be known to the principal. Second, the uncertainty about agent's ability might be interpreted as uncertainty about the quality of the match ,which is similarly unknown to both the agent and the principal. Further, we note that although the analysis begins with a common prior assumption, over the course of time, it is possible that beliefs about ability might diverge due to asymmetric information.

[8]Thus one can interpret ability of the agent as corresponding to his ability to capitalize on opportunities

[9]The private benefit includes benefits such as publicity as well as learning in case of the entrepreneurship example and also takes into account effort cost of implementing projects - thus one can interpret b as the net benefit to the agent from implementing projects.

both the principal and the agent are normalized to 0 each. The principal values successful outcome at $R > c$.[10]

Information*:* In each period, only the agent observes if a good project is available. Given the financing from the principal, the agent has a choice between implementing no project, implementing a bad project or implementing a good project (if available). The principal can observe if a project is chosen in a specific period and also what the outcome of the project is. In particular, success in a project is immediately observed by both the principal and the agent. The quality of the project chosen in case of failure of the project is not observed by the principal (even ex post).

Learning*:* Not implementing a project provides no information regarding the ability of the agent. Suppose the principal expects the agent to implement projects if and only if they are good. In that case, failure leads to a reduction of the principal's belief regarding the ability of the agent. Let α_k denote the probability that the agent is of high ability given k past failures and no success. Then (assuming again that the agent only implements good projects) Bayes' law implies

$$\alpha_k = \frac{(1-\gamma)^k \alpha_0}{(1-\gamma)^k \alpha_0 + (1-\alpha_0)}. \tag{1}$$

Success in a project reveals that the agent is of high ability since only high ability agents can succeed.

We note that it is possible for the beliefs of the agent and the principal to diverge. In particular, if an agent selects to implement a bad project, then his belief will be unchanged following failure. However if the principal expected the agent to implement a project if and

[10]Since only high ability agent can achieve success, R summarizes the future surplus the principal gets from interacting with a high ability agent.

only if it was good and she sees the project fail, then she will reduce her belief regarding the agent's ability.

Contracts: We consider contracting at period zero with full commitment on part of the principal. We restrict attention to contracts in which (i) the agent implements a project if and only if it is good and (ii) payments are conditional only on the number of past failures. Formally, a contract is given by (k, X) where $k \in \{0, 1, ...\}$ is the maximum number of trials the principal is willing to fund and $X = (X_{0k}, X_{1k}, .., X_{sk}, ..X_{k-1k})$ specifies the transfer [11] to be made to the agent. In particular, X_{sk} stands for the transfer to the agent conditional on the agent succeeding after s failures and the contract allowing for a total of k failures. We assume limited liability: X_{sk} cannot be negative. This is not the most general set of contracts. However, the simplifying assumptions on the contract set are designed to bring out in the simplest possible way what the basic economic tension is in the delegation and learning problem. Once the basic tradeoff is clearly modeled, it is easier to explore the robustness of the optimal contract to generalizations of the contract set.

One possible interpretation of the contracts under study is as follows. The agent has no money of his own to fund projects. At the beginning of the game, the principal commits to a line of credit up to an amount kc to be used for undertaking projects where k is a non-negative integer and is a choice variable for the principal. This provides enough funds to try k projects since each project requires c to be implemented. If the agent exhausts the funding without obtaining a success, the game ends. The other contingency where the game ends is when the first success is achieved and the agent is rewarded with a bonus following the success.

Histories: There are two relevant histories to keep track of. One is the public history of

[11] An alternative interpretation for X is given in section 6.

past failures, specifically the number of failures up to period t.[12] The other is the agent's private history including the number of past failures up to t and the quality of projects implemented up to t.[13]

Let Π_k denote the principal's expected payoff at time 0 from a contract which allows for k trials and has the agent implement a project if and only if it is good. The principal's problem is to choose k and $(X_{sk})_{s=0,1,\ldots k-1.}$ at time 0 to maximize her expected payoff Π_k. The agent's strategy at a given point in time is to choose which project (if any) to implement that period as a function of his private history and the projects available at that period. Let $V_k(m, s)$ be the agent's expected payoff after s failures, m of which were good projects, in a k-trial contract[14].

Figure 1 illustrates the game tree for the stage game when both good and bad projects are available and there have been s failures in projects out of which $m \leq s$ were failures in good projects. If $s \geq k$, then the principal does not finance projects and hence the payoff to both the principal and the agent is given by 0 each. If $s < k$, then the principal finances the project. If the agent chooses not to implement a project, then both the principal and the agent get 0 each and the number of failures in projects remains unchanged. If the agent implements a bad project, then the agent gets b and the principal gets $-c$. The number of projects which have failed is given by $s + 1$, while the number of failures in good projects is still given by m. If the agent implements a good project, then it can result in either success or failure. In case the project fails, the agent gets b while the principal gets $-c$. The number

[12]Note that since the contract specifies payments only as function of number of past failures, it's not required to track the order of sequence of failures and non-implementation. This is without loss of generality given the IID assumption regarding the availability of good projects.

[13]The agent's private history also includes availability of projects in past periods, however this does not affect payoff.

[14]If the principal expects the agent to implement a project iff it is good, then s failures corresponds to the principal's belief about the agent's ability to be α_s while m failures in good projects corresponds to the agent's belief about his ability to be α_m. There is thus a one to one map between the number of failures (m, s) and the beliefs (α_m, α_s).

of projects that have failed equals $s + 1$ while the number of good projects that have failed is given by $m + 1$. Since the principal does not observe the quality of the project but only observes failure, she cannot identify if the project implemented was good or bad. If the good project succeeds, the principal pays the agent X_{sk}. Thus the agent's payoff is given by $b + X_{sk}$ while the principal's payoff is given by $R - X_{sk} - c$.

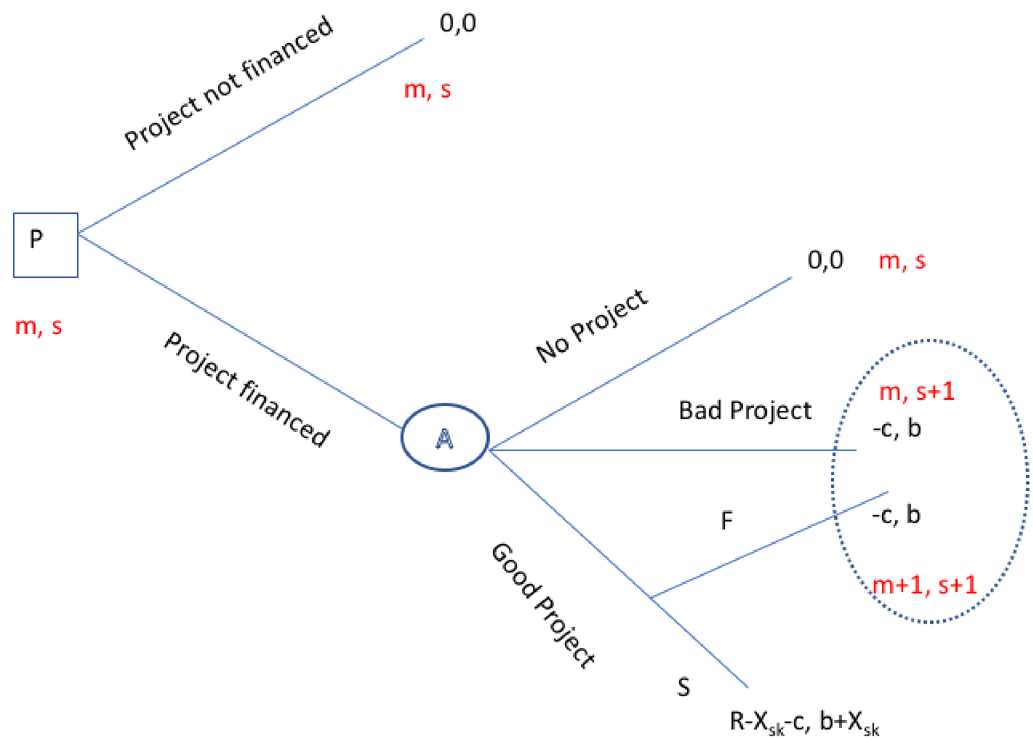

Figure 1: Tree for Stage Game

Notes: The above figure represents the stage game when both good and bad projects are available and there have been s failures in projects out of which $m \leq s$ were failures in good projects. P stands for the principal and A stands for the agent.

2.2 Complete Information Benchmark

In this subsection, we derive the optimal contract when the principal can observe the quality of the projects available each period and write a contract which can include the quality. In this case, the principal implements a project if and only if it is good and keeps experimenting until the point at which her belief falls below a cutoff level. We derive below this cutoff belief.

Let α_k denote the belief regarding the agent upon observing k failures and zero successes. Suppose that there is a good project available. Then if the principal permits the good project to be implemented and stops experimenting if the project fails, her payoff is given by

$$\alpha_k \gamma R - c.$$

In the above expression, $\alpha_k \gamma$ refers to the probability of success in a good project given k failures and zero successes in good projects and R is the payoff to the principal in the event of success. Thus expected surplus from implementing a good project is given by $\alpha_k \gamma R$ and c is the cost of implementing a project.

The principal should thus experiment as long as the above payoff is non-negative, that is till the highest k such that

$$\alpha_k \gamma R \quad \geq \quad c.$$

Assumption 1: Experimentation is initially profitable in the absence of an agency problem:

$$\alpha_0 \gamma R \quad \geq \quad c.$$

This assumption means that without the agency problem, the principal would be willing to experiment at least once at the initial belief.

3 The Special Case with at Most One or Two Trials

This section illustrates some basic insights and tradeoffs in the special case where first there is only one trial and second where there may be up to two trials.

3.1 One Trial Contract

In this case, the agent gets only one shot at implementing a project. For the contract that allows for one trial, we need to determine the optimal bonus X_{01} that incentivizes the agent to implement the project if and only if the project is good. The incentive compatibility condition for not implementing a project over choosing a bad project is given by

$$\delta V_1(0,0) \quad \geq \quad b. \tag{2}$$

The equation says that the payoff to the agent from not implementing a project has to be greater than that of selecting the bad project. The payoff from not implementing a project is given by $\delta V_1(0,0)$. It refers to the observation that if the agent chooses to not implement a project, then he gets 0 this period and the next period utility for the agent is still given by $V_1(0,0)$ since the the number of failures are unchanged if the agent selects not to implement a project. If the agent selects the bad project, then he gets b this period but the project is sure to fail and since the contract only allows for one trial, his continuation payoff is 0.

The incentive compatibility condition for choosing the good project if it is available is given by

$$b + \alpha_0 \gamma X_{01} \quad \geq \quad \max(\delta V_1(0,0), b). \tag{3}$$

Given equation (2), we can simplify as

$$b + \alpha_0 \gamma X_{01} \geq \delta V_1(0,0). \tag{4}$$

The left side stands for the expected payoff to the agent if he selects a good project. It consists of the current gain b that the agent makes if he implements a project and the expected bonus in case of success. Since the project is good and the belief that the agent is of high ability is given by α_0, the probability of success is given by $\alpha_0\gamma$. In case of success, the agent is rewarded by the bonus X_{01} as stated in the contract. The contract allows for only one trial; hence if the agent fails, the principal chooses to stop experimenting in which case the agent receives 0. The term on the right side refers to the payoff from not implementing a project which is same as before.

The agent's ex-ante value in such an incentive compatible contract is given by

$$\begin{aligned} V_1(0,0) &= p(b + \alpha_0\gamma X_{01}) + (1-p)\delta V_1(0,0) \\ &= \frac{p}{1-\delta(1-p)}(b + \alpha_0\gamma X_{01}). \\ &= \theta(b + \alpha_0\gamma X_{01}) \end{aligned} \tag{5}$$

where p denotes the probability a good project is available in a period and $\theta \equiv \frac{p}{1-\delta(1-p)}$. Since both p and δ lie between 0 and 1, we get $0 < \theta < 1$.

We observe that incentive compatibility for the good project is always satisfied since $\delta < 1$ and the expected payoff from implementing a project is non-negative[15]. Hence we only need to make sure that X_{01} is high enough so that incentive compatibility condition for the

[15]From equation (5), we obtain $V_1(0,0) = \theta(b + \alpha_0\gamma X_{01})$. Inserting this in equation (4), the right hand side equals $\delta\theta(b + \alpha_0\gamma X_{01})$. Since $0 < \delta, \theta < 1$, equation (4) is satisfied.

bad project is satisfied. Plugging in the value of $V_1(0,0)$ and solving for X_{01} we obtain,

$$X_{01} \geq \frac{b(1-\delta)}{\delta\alpha_0\gamma p}. \tag{6}$$

The principal's expected payoff from this contract is given by Π_1 which satisfies

$$\Pi_1 = \theta[\alpha_0\gamma(R - X_{01}) - c].$$

The term $R - X_{01}$ represents the payoff to the principal in case of success while c stands for the cost of implementing the project. Since the contract allows for only one failure, one failure ends the experimentation. As the bonus payments enter negatively in the principal's profit, she won't pay the agent more than required and hence inequality (4) is satisfied with an equality. Thus we get

$$X_{01} = \frac{b(1-\delta)}{\delta\alpha_0\gamma p}. \tag{7}$$

We thus observe that X_{01} is an increasing function of b and a decreasing function of $\delta, \alpha_0, \gamma, p$. The purpose of having $X_{01} > 0$ is to ensure that if the agent comes across a bad project, the expected reward from foregoing on the bad project and waiting for a good project to come along is high enough that he is willing to not implement the bad project. The cost of passing up on the bad project at hand is the private benefit b. Hence higher is the b, greater the incentive needs to be for the agent to pass up on that in the current period. Since the agent has to wait till at least the next period to see if a good project comes along, the more impatient an agent is, higher needs to be the bonus from succeeding in a good project. The bonus is only paid out in the event of success in the good project and hence it is decreasing in $\alpha_0\gamma$, the probability of success of the good project. Finally, the lower the value of p, the more the agent needs to wait for a good project to come along and hence the reward for waiting has to be higher.

The corresponding expected payoff to the agent from accepting the contract is given by

$$V_1(0,0) \;=\; \frac{b}{\delta}.$$

The principal should prefer to offer this contract over not experimenting at all if and only if $\Pi_1 \geq 0$ which gives us:

$$\alpha_0 \gamma R \;\geq\; c + \frac{b(1-\delta)}{\delta p}.$$

Assumption 2:

$$\alpha_0 \gamma R \;>\; c + \frac{b(1-\delta)}{\delta p}. \tag{8}$$

This inequality says that the principal will want to experiment at least once even in the second best.

3.2 Two Trials Contract

In this case, the agent gets at most two shots at implementing projects. We first consider what happens in case the first trial results in failure. If the first trial fails, there is only one more failure permitted in the contract. Hence the analysis is similar to the analysis for one failure contract considered above. Since the contract requires the agent to implement a project if and only if it is good and on path the belief of the agent is α_1 after the first failure, we obtain

$$X_{12} \geq \frac{b(1-\delta)}{\delta\alpha_1\gamma p}. \tag{9}$$

We observe that the bonus offered to incentivize the agent in the last opportunity has to be higher in the contract with two trials than in the contract with one trial that is $X_{12} > X_{01}$. This is because the agent's belief about his ability is lower and hence he needs a higher incentive to wait for the good project.

In order to determine X_{02}, we consider the incentive compatibility conditions prior to first failure. The incentive compatibility condition for selecting the good project given 0 failures is now given by

$$b + \alpha_0\gamma X_{02} + (1-\alpha_0\gamma)\delta V_2(1,1) \geq \delta V_2(0,0). \tag{10}$$

Since the principal does not stop experimenting immediately after a failure but allows the agent to continue to experimenting, the agent's payoff upon failure is given by $\delta V_2(1,1)$ and not 0 as before.

The incentive compatibility condition for rejecting the bad project gives us

$$\delta V_2(0,0) \geq b + \delta V_2(0,1). \tag{11}$$

Unlike the one failure contract, failure in a project in this case does not stop experimentation. The agent does not update his beliefs about himself after the expected failure but the principal's belief declines to α_1 (as implementing the bad project is "off path"; that is, the principal was expecting the agent to implement only good projects). We note that even if the agent deviates from the principal's prescribed strategy after 0 failures to implement a bad project, he will choose to implement a project iff good in the second trial. This follows from verifying that the two incentive compatibility conditions - (i) $b + \alpha_0\gamma X_{12} \geq \delta V_2(0,1)$ and (ii) $\delta V_2(0,1) \geq b$ are satisfied[16]. The agent's value from the contract in such a case is given by

$$V_2(0,1) \;=\; \theta[b + \alpha_0\gamma X_{12}]$$

The agent's ex-ante value (on path) is given by

$$V_2(0,0) \;=\; \theta[b + \alpha_0\gamma X_{02} + (1-\alpha_0\gamma)\delta V_2(1,1)].$$

Once again the incentive compatibility for the good project is satisfied since $\delta < 1$ and the expected payoff from implementing the project is non-negative. Thus we only need to make sure that X_{02} is high enough so that incentive compatibility condition for the bad project is satisfied. Plugging in the value of $V_2(0,0)$ and solving for X_{02} we get,

$$X_{02} \;\geq\; \underbrace{\frac{b}{\theta\alpha_0\gamma}[\frac{1}{\delta} - (1-\alpha_0\gamma)\delta\theta^2]}_{>0} + X_{12}\underbrace{[1-\delta\theta(1-\gamma)]}_{>0} \tag{12}$$

We thus observe that X_{02} is an increasing function of X_{12}.

The principal's expected payoff from offering a contract which allows for two trials is given by Π_2 which satisfies

$$\begin{aligned}\Pi_2 \;=\; & \theta[\alpha_0\gamma(R - X_{02}) - c] \\ & +\delta\theta^2(1-\alpha_0\gamma)[\alpha_1\gamma(R - X_{12}) - c].\end{aligned}$$

Since both X_{02} and X_{12} enter negatively in the expression for expected payoff and X_{02} is increasing in X_{12}, the principal will try to minimize these two as much as possible. Hence

[16] This is discussed in more detail in Section 4.

both equations (7) and (10) hold with equality and we obtain,

$$X_{12} = \frac{b(1-\delta)}{\delta\alpha_1\gamma p}. \tag{13}$$

$$X_{02} = \frac{b(1-\delta)}{\delta\alpha_0\gamma p} + \frac{b(1-\delta)}{\delta\alpha_1\gamma p} + b. \tag{14}$$

It's useful to think about the individual terms in the above expression. The first term plays a similar role as the term in equation (4) - it provides incentives to forego on the bad project in favor of waiting for a good project to come along. However in equation (14), there are now two additional terms - these refer to the fact that in the contract with two failures there are additional benefits to selecting a bad project when there is another opportunity still remaining. If the agent selects a bad project, he knows for sure that the game will not end this period - since the project is sure to fail - and hence gives the agent an opportunity to earn further rent. There are two sources of this additional rent. First, the agent gets to implement another project which gives him a benefit of $b > 0$. Second, the agent has the opportunity to gain an additional rent because his belief is higher than the belief which the principal had in mind while designing the bonus for the next project - we can see this from

$$V_2(0,1) = \theta[b + \alpha_0\gamma X_{12}] = \theta[b + \frac{\alpha_0}{\alpha_1}\frac{b(1-\delta)}{\delta p}] > \theta[b + \frac{b(1-\delta)}{\delta p}] = V_2(1,1).$$

We also see that $X_{02} > X_{12}$ - that is the contract has to be front-loaded. While comparing X_{02} and X_{12} we see that the agent is more pessimistic about his ability upon implementing a good project and failing - hence he has to be possibly provided a greater incentive in order to make sure he waits for the good project. On the other hand, the agent has to be provided additional incentives in the first attempt to compensate him to forego the possible rents from taking up the second project as outlined in the previous paragraph. What $X_{02} > X_{12}$ says is that the second effect dominates and hence the contract is front-loaded. We note that this contrasts with some of the existing results in the literature. For example, Halac, Kartik and Liu (2016) found instances where contracts have bonuses which are increasing as the agent gets more pessimistic. The main difference in our model is that the agent gets a benefit each time a project is undertaken and hence the contract has to compensate the agent for the loss in continuation value in order to incentivize him to implement only good projects.

It's also useful to compare X_{02} with the bonus X_{02}^V that the principal would have to pay to the agent in the scenario the principal could verify the quality of the project implemented before giving permission to go ahead with the second trial and could commit to firing the agent in case it was discovered that he had selected a bad project. In this case the the bonus[17] can be obtained as

$$X_{02}^V = \frac{b(1-\delta)}{\delta\alpha_0\gamma p}$$

Thus if the principal could verify the project quality ex-post and commit to firing the agent for selecting selecting the bad project, the contract becomes back-loaded that is $X_{02}^V < X_{12}$.

We also note that $X_{02} > X_{01}$, that is increasing the number of trials implies that earlier success have to be rewarded more in the contract which has higher number of trials. This is because the agent has to be compensated for greater losses in rents in the contract with higher number of trials.

If we compare Π_2 with Π_1, we see that the principal faces benefits and costs in moving

[17] This is also the bonus that the principal would pay to an agent if he can costlessly replace the agent with another agent upon failure in a project. In this case though, ability is not agent-specific, but is more about the quality of the idea that is being assessed through projects.

from a contract with 1 trial to 2 trials. The change in the expected payoff can be decomposed as:

$$\Pi_2 - \Pi_1 = \underbrace{\delta\theta^2(1-\alpha_0\gamma)[\alpha_1\gamma(R - X_{12}) - c]}_{benefit} - \underbrace{\theta\alpha_0\gamma(X_{02} - X_{01})}_{cost}$$

The additional benefit captures the scenario that the high ability agent might fail while attempting a good project on the first attempt which happens with probability $(1-\alpha_0\gamma)$ but allows for the possibility that the agent succeeds on the second attempt. The cost reflects the higher bonus that has to be paid to the agent.

If we compare it to the case with one failure we see that that the principal gets a lower payoff if the agent succeeds in the first attempt since $X_{02} > X_{01}$. Thus the main tradeoff to the principal is increasing the number of experiments funded leads to more accurate information about the ability of the agent but has to be paid for not only in terms of more cost of experimentation but also in higher rents to the agent in case of earlier success.

4 Optimal Contract

In this section, we examine the properties of the optimal contract that incentivizes the agent to implement the project if and only if it is a good project.

We can decompose the problem into a two step procedure: First, given a maximum number k of trials that the principal is willing to fund, what should the optimal bonus scheme be in order for the agent to choose the project if and only if it is a good project? Having found the optimal bonus scheme, we determine the number of trials the principal is willing to fund.

4.1 Optimal Bonus

Definition: Given a maximum number of trials k that the principal is willing to fund, we say that the bonus scheme $(X_{sk})_{s=0,1\ldots k-1}$ is incentive compatible if under such a bonus scheme the agent chooses to implement projects if and only if they are good projects. We define an optimal contract as a contract that is incentive compatible and maximizes the principal's expected payoff.

Let $(X_{sk})_{s=0,1\ldots k-1}$ be a incentive compatible bonus scheme. Let $\Pi_{s,k}$ denote principal's expected payoff from such a contract when $s < k$ failures and zero successes in good projects have taken place. Then $\Pi_{s,k}$ satisfies the following the recurrence relation :

$$\Pi_{s,k} = p[\alpha_s\gamma(R - X_{s,k}) + (1 - \alpha_s\gamma)\delta\Pi_{s+1,k} - c] + (1 - p)\delta\Pi_{s,k}$$

With probability p, a good project becomes available and is implemented. This leads to an expected profit of $\alpha_s\gamma(R - X_{sk}) + (1 - \alpha_s\gamma)\delta\Pi_{s+1,k} - c$. Given that the bonus scheme is incentive compatible, all the earlier failures were in good projects and hence the probability that the agent is high ability is given by α_s from equation (1). Thus the probability of success in the good project is given by $\alpha_s\gamma$. In case of a success, the principal gets $R - X_{sk}$ since the contracts specifies X_{sk} as the bonus to be paid in such a situation. In case of a failure which happens with probability $(1 - \alpha_s\gamma)$, the future payoff is given by $\delta\Pi_{s+1,k}$. Finally c stands for the cost of implementing the project. With probability $1 - p$, the good project is not available and thus a project is not implemented. Hence we move on to the next period and the profit for the principal is summarized by $\delta\Pi_{s,k}$.

The above recurrence relation can be further simplified to yield

$$\Pi_{s,k} = \theta[\alpha_s\gamma(R - X_{sk}) + (1 - \alpha_s\gamma)\delta\Pi_{s+1,k} - c]$$

where $\theta \equiv \frac{p}{1-\delta(1-p)}$. Thus the overall expected profit from offering a contract which tolerates k failures is given by $\Pi_{0,k} \equiv \Pi_k$ where

$$\begin{aligned}
\Pi_k &= \theta[\alpha_0\gamma(R - X_{0k}) + (1-\alpha_0\gamma)\delta\Pi_{1,k} - c] \\
&= \theta[\alpha_0\gamma(R - X_{0k}) - c] + \theta(1-\alpha_0\gamma)\delta\Pi_{1,k} \\
&= \theta[\alpha_0\gamma(R - X_{0k}) - c] + \\
&\quad \theta^2\delta(1-\alpha_0\gamma)(\alpha_1\gamma(R - X_{1k}) + (1-\alpha_1\gamma)\delta\Pi_{2,k} - c) \\
&= \theta(\alpha_0\gamma(R - X_{0k}) - c) + \sum_{s=1}^{k-1}\theta^{s+1}\delta^s(\prod_{m=0}^{s-1}(1-\alpha_m\gamma))(\alpha_s\gamma(R - X_{sk}) - c)
\end{aligned}$$

Given k, the principal's profit maximization problem is to choose $(X_{sk})_{s=0,1,...,k-1}$and $(V_k(m,s))_{m=0}^{s}$ to maximize Π_k subject to the following constraints: for each $s = 0, ..., k-1$,

$$b + \alpha_m\gamma X_{sk} + (1-\alpha_m\gamma)\delta V_k(m+1, s+1) \geq \delta V_k(m,s) \qquad \text{(IC-G)}$$

$$b + \delta V_k(m, s+1) \leq \delta V_k(m,s) \qquad \text{(IC-B)}$$

$$X_{sk} \geq 0 \qquad \text{(LL)}$$

where $V_k(m,s)$ is defined by:

$$
\begin{aligned}
V_k(m,s) &= \max_{1_{Gms},1_{BGms},1_{Bms},1_{Gms}+1_{BGms}\leq 1} && \{p[1_{Gms}(b+\alpha_m\gamma X_{sk} \\
& && +(1-\alpha_m\gamma)\delta V_k(m+1,s+1)) \\
& && +1_{BGms}(1-1_{Gms})(b+\delta V_k(m,s+1)) \\
& && +(1-1_{BGms})(1-1_{Gms})\delta V_k(m,s)] \\
& && +(1-p)[1_{Bms}(b+\delta V_k(m,s+1)) \\
& && +(1-1_{Bms})\delta V_k(m,s)]\}
\end{aligned}
$$

where 1_{Gms} is an indicator function which takes value $= 1$ if the agent selects the good project (after s public failures of projects, of which m were good) if it is available and 0 otherwise. Similarly 1_{BGms} stands for the indicator function for the agent's choice regarding an implementation of bad project if a good project is available ((after s public failures of projects, of which m were good) while 1_{Bms} stands for the indicator function for the agent's choice regarding implementation of a bad project (after s public failures of projects, of which m were good) if a good project is not available.

Our first result deals with the question of how should the principal set (X_{sk}) to maximize the expected profit from such a contract.

Proposition 1: Suppose the principal's optimal contract funds up to k trials. Then bonuses $(X_{sk})_{s=0,1,\ldots,k-1}$ in this contract are given by

$$X_{sk} = (k-1-s)b + \sum_{m=s}^{k-1} \frac{b(1-\delta)}{\delta p \gamma \alpha_m}.$$

Proof: See the appendix

Sketch of the proof

The proof is divided into the following steps. Instead of the profit maximization problem, we focus on the equivalent cost minimization problem.

Step One: We first consider a relaxed problem by restricting agent's off path strategies to have only one-period deviations - that is the agent can only deviate once (by either choosing not to implement a project when a good project is available or by implementing a bad project) but from then on will choose to implement projects if and only if they are good projects. Since the bonus schemes are such that they act as incentives against all deviations, it has to be true that they prevent the agent from these types of deviations. We can thus write the relaxed problem as

$$\min_{(X_{sk})_{s=0}^{k-1}} \{\theta\alpha_0\gamma X_{0k} + \sum_{s=1}^{k-1} \theta^{s+1}\delta^s[\prod_{m=0}^{s-1}(1-\alpha_m\gamma)][\alpha_s\gamma X_{sk}]\}$$

subject to for each $s = 0, ..., k-1$,

$$b + \alpha_s\gamma X_s + (1-\alpha_s\gamma)\delta V_k^T(s+1, s+1) \geq \delta V_k^T(s, s) \quad \text{(IC-G-O-s)}$$

$$b + \delta V_k^T(s, s+1) \leq \delta V_k^T(s, s) \quad \text{(IC-B-O-s)}$$

$$X_{sk} \geq 0 \quad \text{(LL)}$$

where

$$V_k^T(s,s) = \theta(b+\alpha_s\gamma X_{sk}) + \sum_{m=s+1}^{k-1} \theta^{m+1-s}\delta^{m-s}[\prod_{n=s}^{m-1}(1-\alpha_n\gamma)][b+\alpha_m\gamma X_{mk}]$$

and

$$V_k^T(s,s+1) = \theta(b+\alpha_s\gamma X_{s+1k}) + \sum_{m=s+1}^{k-2} \theta^{m+1-s}\delta^{m-s}[\prod_{n=s}^{m-1}(1-\alpha_n\gamma)][b+\alpha_m\gamma X_{m+1k}]$$

Step Two: We then show that IC-G-O-s are satisfied. To see this, we observe that $V_k^T(s,s)$ can be rewritten as

$$V_k^T(s,s) = \theta(b+\alpha_s\gamma X_s + (1-\alpha_s\gamma)\delta V_k^T(s+1,s+1))$$

Thus we can rewrite the IC-G-O-s as

$$b+\alpha_s\gamma X_s + (1-\alpha_s\gamma)\delta V_k^T(s+1,s+1) \geq \delta\theta(b+\alpha_s\gamma X_s + (1-\alpha_s\gamma)\delta V_k^T(s+1,s+1))$$

which is always satisfied since $b+\alpha_s\gamma X_s+(1-\alpha_s\gamma)\delta V_k^T(s+1,s+1) > 0$ and $0 < \delta, \theta < 1$.

Step Three: Next, if the only off path strategies available to the agent are these one-period deviations, then all the IC-B-O-s need to hold with equality, otherwise the principal can decrease bonuses without affecting incentives following s failures and before to increase profit[18].

Step Four: Based on the IC-B-O-s holding with equality, we obtain a difference equation

[18]It is possible to decrease bonuses without violating limited liability conditions since one can show that $X_{sk} > 0$, which follows from IC-B-O-s and induction - the details are discussed in the appendix.

linking X_{sk} and X_{s+1k}:

$$X_{sk} = \frac{b(1-\delta)}{\delta\gamma\alpha_s p} + X_{s+1k} + b$$

along with the boundary condition:

$$X_{k-1k} = \frac{b(1-\delta)}{\delta\gamma\alpha_{k-1} p}$$

This gives us a solution for X_{sk} as stated in the proposition.

Step Five: We show that the X_{sk} we found by restricting the agent's off-path strategy are enough to deter the agent from more complex off-path strategies involving multiple deviations. Intuitively, the contract in the relaxed problem ensured that the agent has no incentives to deviate if never deviated. The agent's private belief is either the same as the public belief (if he deviates by not implementing a project when a good project is available) or higher (if he deviates by selecting a bad project). Hence we can verify deviating is even less attractive to the agent if he has deviated before.

Proposition 1 lends itself to the following two corollaries:

Corollary 1: Bonuses are front-loaded i.e $X_{0k} > X_{1k} > ... > X_{k-1k}$.

The intuition is that earlier bonuses need to compensate the agent for giving up the rents that he could have got from future projects as well as rents due to the possibility of divergence between the private belief of the agent and the belief based on public history.

Corollary 2: Increasing the number of failures allowed increases the bonus needed to incentivize the agent at each stage: $X_{sk} > X_{sk'}$ for $k > k'$ for $s = 0, 1...k' - 1$.

The intuition follows from observing that an increase in the number of trials implies that the agent has an opportunity to get greater private benefits by implementing more projects as well as earn higher rents by causing a greater divergence between public and private beliefs. Thus the agent has to be compensated for a greater potential loss of continuation rents for selecting good projects when there is an increase in number of maximum failures allowed.

4.2 Optimal Number of Trials

Having found the optimal bonus scheme, we move on to examine the question of how should the principal decide on the optimal number of trials. To understand the determinants, it's useful to decompose the impact on expected payoff of the principal as a result of a change in the number of trials. The change in payoff for the principal if he decides to increase the number of trials from k to $k + 1$ is given by

$$\begin{aligned} \triangle\Pi_k &= \theta^{k+1}\delta^k[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma(R - X_{kk+1}) - c] \\ &\quad - \sum_{s=1}^{k}\theta^s\delta^{s-1}\alpha_0(1-\gamma)^{s-1}\gamma(X_{s-1\,k+1} - X_{s-1\,k}) \\ &\equiv MB_k^{SB} - MC_k^{SB} \end{aligned}$$

We can decompose the total change in the expected payoff of the principal into two parts: the "marginal benefit" and the "marginal cost". We define and expand on the terms below.

Increasing the number of trials from k to $k + 1$ has two consequences for the principal's expected payoff - first, there is an additional opportunity to succeed in case the first k trials result in failure and second, the bonuses for success in the first k trials have to be altered as a consequence of corollary 2.

Since the number of trials has gone up from k to $k+1$, there is now an additional opportunity to experiment. The "marginal benefit" refers to the impact on the expected payoff due to the principal having one additional chance of experimentation, holding fixed the bonus to be paid in case of success in the first k trials. We note that the additional trial is of use only if the first k trials have resulted in failure. For $k \geq 1$, the expected payoff from the additional opportunity is given by

$$MB_k^{SB} \equiv \theta^{k+1}\delta^k[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma(R-X_{kk+1})-c]$$

We can decompose this expression into two parts - $\prod_{m=0}^{k-1}(1-\alpha_m\gamma)$ refers to the probability of no success in the first k trials while $\theta^{k+1}\delta^k[\alpha_k\gamma(R-X_{kk+1})-c]$ refers to the expected payoff for success in the $k+1^{th}$ trial. $MB_0^{SB}$is given by $\theta[\alpha_0\gamma(R-X_{01})-c]$.

Lemma 1: The "marginal benefit" of experimentation is decreasing in the maximum number of failures tolerated by the principal, that is MB_k^{SB} is a decreasing function of k.

Proof: See the appendix

The intuition is that not only does the new opportunity present itself much later (which is reflected in the terms $\theta^{k+1}\delta^k$), but it is also less likely to present itself - the probability is given by $\prod_{m=0}^{k-1}(1-\alpha_m\gamma) = \{1-\alpha_0+\alpha_0(1-\gamma)^k\}$ - and also when it presents itself the expected payoff $(\alpha_k\gamma(R-X_{kk+1})-c)$ is decreasing in k as well since the probability of success $\alpha_k\gamma$ is lower and the principal also needs to pay a higher bonus X_{kk+1} to incentivize the agent.

The "marginal cost" captures the fact that increasing the number of trials permitted results in increasing the bonus that has to be promised to the agent in case of success after $0, 1, ...k-1$ failures. This observation follows from corollary 2. The "marginal cost"[19] for $k \geq 1$ is given by

$$MC_k^{SB} \equiv \sum_{s=1}^{k} \theta^s \delta^{s-1} \alpha_0 (1-\gamma)^{s-1} \gamma (X_{s-1k+1} - X_{s-1k})$$

In the above expression, $\alpha_0(1-\gamma)^{s-1}\gamma$ refers to the probability of success in the s^{th} trial, while $\theta^s\delta^{s-1}(X_{s-1k+1} - X_{s-1k})$ refers to (discounted) value of increased bonus. We also define $MC_0^{SB} \equiv 0$.

Using the result for the optimal bonuses from proposition 1, we can rewrite the "marginal cost" for an incentive compatible contract as

$$MC_k^{SB} = \sum_{s=1}^{k} \theta^s \delta^{s-1} \alpha_0 (1-\gamma)^{s-1} \gamma (b + \frac{b(1-\delta)}{\delta \alpha_k \gamma p})$$

Lemma 2: The "marginal cost" of experimentation is increasing in the maximum number of failures tolerated by the principal, that is MC_k^{SB} is increasing function of k.

Proof: See the appendix

The intuition is that a higher value of k implies a lower value of α_k which results in a higher increase in bonus to be paid in the event of earlier success as $X_{s-1k+1} - X_{s-1k} = b + \frac{b(1-\delta)}{\delta \alpha_k \gamma p}$ as well as there being a higher probability of earlier success since $\sum_{s=1}^{k} \theta^s \delta^{s-1} \alpha_0 (1-\gamma)^{s-1} \gamma$ is increasing in k as well.

[19]One could decompose the effect on expected profit due to an increase in the number of trials in different ways. However it is instructive for the analysis to have the cost of financing a project c be subtracted from the "marginal benefit", as opposed to including it as part of "marginal cost".

Once we have the decomposition of changes in expected payoff of the principal as a result of changing the number of trials allowed, we can characterize the optimal number of trials that the principal will optimally allow. The change in expected payoff due to a change in the maximum number of trials can be viewed as the difference of the "marginal benefit" and the "marginal cost". The change in expected payoff is positive as long as the "marginal benefit" exceeds the "marginal cost" and thus the principal should choose the largest number of trial for which the "marginal benefit" exceeds the "marginal cost". This is also illustrated in Figure 2 below.

Proposition 2: The optimal number of trials is unique and given by the highest k for which $MB_k^{SB} \geq MC_k^{SB}$

We can also compare the optimal number of trials in the complete information benchmark and the second best. In the complete information case, there is no bonus to be paid and hence the "marginal cost" as defined above equals 0 for any number of trials decided upon by the principal[20]. We thus have $MC_k^{CI} = 0$ for any k. The "marginal benefit" of an additional trial in the complete information benchmark is given by

$$MB_k^{CI} \equiv \theta^{k+1}\delta^k[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma R - c]$$

The "marginal benefit" is higher in the complete information benchmark as compared to the second best. Hence the principal should experiment more in the complete information as compared to the situation in which the agent has to be incentivized through bonuses. This

[20]Recall that the cost of financing a project c is subtracted from the marginal benefit in the decomposition described above.

discussion is summarized in the following proposition.

Proposition 3: The second best allows for an inefficiently low number of trials compared to the complete information benchmark.

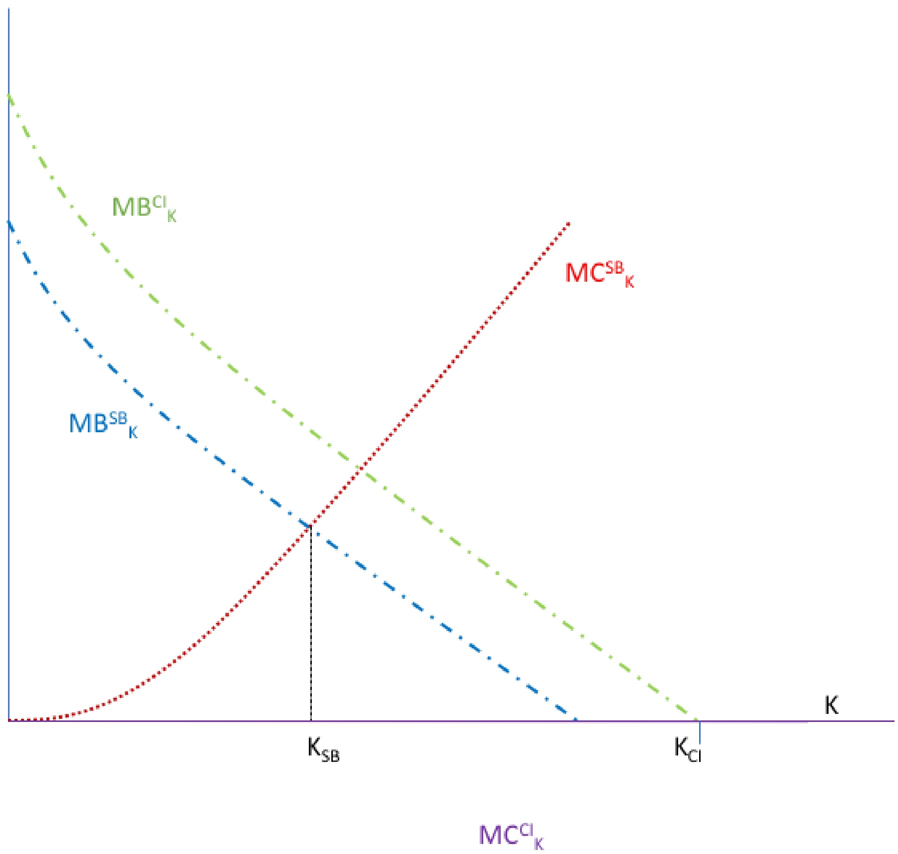

Figure 2: Optimal number of trials
Notes: MB_K^{CI} and MB_K^{SB} stand for "marginal benefit" for the complete information and second best cases respectively; MC_K^{CI} and MC_K^{SB} stand for "marginal cost" for the complete information and second best cases respectively; K_{CI} and K_{SB} denote the optimal number of trials in the complete information and second best cases respectively and K stands for the number of trials.

5 Comparative Statics

In this section, we provide comparative statics results on the number of trials and the principal's expected payoff as a function of parameters.

Proposition 4: The principal's second best expected payoff as well as the optimal number of trials are increasing in R and α_0 and decreasing in c.

Comparative statics with respect to α_0

To understand how a change in α_0 impacts the optimal number of trials, we look at how it impacts the MB_k^{SB} and MC_k^{SB}. We note that MB_k^{SB} is an increasing function of α_0 (all proofs are in the appendix) while it is possible for MC_k^{SB} to be either an increasing or decreasing function α_0. The impact on the "marginal cost" is driven through two channels - holding fixed the number of trials - an increase in α_0 leads to a reduction in bonus paid when success happens after a specific number of failures. However it is also more likely that the agent succeeds earlier, which combined with the front-loading of bonuses imply that the principal could end up paying more. Hence the impact on MC_k^{SB} is ambiguous. Thus it might seem possible that as a result of increase in α_0, the increase in "marginal cost" is so high that the principal might end up reducing the number of experiments he wants to perform. However as shown in the Appendix, an increase in the prior is always leads to an increase in the optimal number of trials.

The effect on expected payoff is unambiguous as well - holding fixed the number of trials, it can be shown that expected payoff of the principal increases as α_0 increases. Since the principal is free to vary the number of trials (which includes the option of not changing the number of trials), her expected payoff is going to be higher in situations when there is an increase in α_0.

Comparative statics with respect to c:

An increase in c leads to a reduction in the "marginal benefit" but has no effect on "marginal cost". Hence the number of trials permitted is going to be weakly lower. Holding fixed the number of trials, expected payoff is decreasing in c and hence an increase in c leads to a reduction in expected payoff.

Comparative statics with respect to R:

An increase in R leads to a increase in the "marginal benefit" but has no effect on "marginal cost". Hence the number of trials permitted is going to be weakly higher. Expected payoff is going to increase following an argument similar to that for the α_0 case.

6 Discussion

6.1 Connecting Predictions with Empirics

The model developed can be applied to venture capital industry. We can view X_{sk} as a measure of cash-flow rights[21] for the entrepreneur upon success. Corollary 1 suggests that the cash-flow rights for the entrepreneurs are a decreasing function of the number of past failures. Kaplan and Strömberg (2003) find evidence that founders' cash flow rights decline over financing rounds and increase with firm performance. They suggest that the increase in VC cash flow rights over financing rounds is consistent with the VC demanding more equity as compensation for providing additional funding. Our model provides an alternative explanation based on incentive theory for reasons why founders' cash-flow rights decline over financing rounds as well as when firm's performance becomes worse.

Our model also has some implications for the structure of anti-dilution provisions which

[21] Cash-flow rights for entrepreneurs are defined as the fraction of a portfolio company's equity value that entrepreneurs have a claim to.

protect previous investors during "down rounds". [22] Anti-dilution provisions are quite common (see for e.g., Kaplan and Strömberg 2003; Gompers, Gornall, Kaplan, and Strebulaev 2019) and are meant to protect the investors against future financing rounds at a lower valuation than the valuation of the current (protected) round. Typically these come at the cost of reduced equity shares for the founders during down rounds and are often associated with loss of motivation on part of the founders. One can interpret the optimal bonuses identified in proposition 1 as a measure of the maximum amount of equity dilution for the entrepreneurs per each round that is consistent with still keeping entrepreneurs incentivized to act in the investor's interest.

The result that an increase in the prior about the entrepreneur's ability is associated with greater financing is consistent with the findings in the empirical literature on venture capital financing which suggests that entrepreneurs who have succeeded in the past are likely to get better deals (Gompers, Kovner, Lerner and Scharfstein 2010). The empirical evidence regarding the effect of c on financing of experimentation is mixed. Recent research (Kerr, Nanda and Rhodes-Kropf 2014; Ewens, Nanda, and Rhodes-Kropf forthcoming) suggests that the main impact of a reduction in c has been in increasing the number of entrepreneurs financed. However investors have reduced the amount of funding to individual entrepreneurs at the initial stage and now wait for more information about future prospects of the investment before committing more resources.

6.2 Non-Monetary Rewards

Our model has so far interpreted X as monetary payments made from the principal to the agent. However in a lot of settings, especially within organizations, ability to exchange money is often limited[23]. Similarly, founders are often rewarded for success not via monetary

[22] A down round is defined as a financing round with a lower share price than the previous round.
[23] The restriction on the use of monetary rewards is a common feature in the delegation literature.

bonuses or cash flow rights but via greater control rights. To capture this in our model, we can also interpret X in our model as promised continuation utilities instead of monetary bonuses. Let $f(X)$ denote the cost to the principal of providing continuation utility X to the agent. Thus now success after s failures results in the agent receiving X_{sk} as before but the principal's payoff is given by $R - f(X_{sk})$. If we assume that $f(X)$ is an increasing convex function of X, then the expression obtained for X_{sk} in proposition 1 remains unchanged. Further the results for the optimal trials as well as the comparative statics results too remain qualitatively similar. Thus our model can be widely applied to settings even where monetary rewards are not available.

6.3 Private Observability and Disclosure

In our model, success in a project was immediately observed by the principal. Suppose instead that the outcome in a project is privately observed by the agent but can be verifiably disclosed. However if success is not immediately disclosed, then they are lost. Further, assume that the principal's payoff from project success obtains here only when the agent discloses it. Then one question that might be of interest is under the optimal contract found above, does the agent have enough incentive to disclose the success? The answer is yes, and one can see it in the context of the two trial example. Suppose the agent implements the first trial in period t and obtains a success. In case he reveals the success, he gets a payoff of $b + X_{02}$. However, if he chooses to hide the success, then he moves on to the second trial. Having received success, the agent knows that he is a high ability type for sure while the belief about his ability based on public history is given by α_1. Hence following the logic for the two trial case, he will indeed choose to wait for the good project to come before choosing to implement a project. His payoff in this case is given by $b + \theta(b + \gamma X_{12})$. Since

$X_{02} > X_{12} + b$, we obtain

$$\begin{aligned} b + X_{02} &> b + b + X_{12} \\ &> b + \theta(b + \gamma X_{12}). \end{aligned}$$

Hence the agent will choose to disclose success as soon as he obtains one.

7 Conclusion

This article studied a dynamic principal-agent model for experimentation in which the agent is financed to work on projects and the principal learns about the agent's ability through observing his performances in the projects. Performance also depends on the quality of the projects implemented; this quality is private information for the agent who is biased towards implementation. We identified the sources of rents received by the agent in this setting and showed that the optimal bonus structure has payments for success decreasing in the number of past failures. The optimal amount of funding to be made available for the agent for implementing projects is determined by comparing the benefits of higher number of opportunities, which reduces the probability that the agent was of high ability but failed due to a lack of sufficient opportunities, and the higher rents to be paid to the agent as a consequence of increasing the number of opportunities.

There are some questions related to the issues analyzed in the article that may be of interest for future research. One possibility is to analyze more general reward structures, for instance by allowing the principal to contract on a richer set of variables such as time or periods in which no project is implemented. Another interesting question to study is what happens in the absence of commitment power on behalf of the principal. Finally, it could also be interesting to study the dynamics of a multi-stage relationship where each stage requires a success - in which case performance in a stage has implications for the incentive structure in later stages. These remain for future research.

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Appendix: Proofs

Proof of Proposition 1:

Fix k, the maximum number of failures permitted. We are interested in characterizing the bonus scheme $(X_{sk})_{s=0,1,\ldots k-1}$ that maximizes the principal’s profit and also ensures that the agent chooses to implement the project if and only if it is a good project.

The proof is divided into the following steps. We first study a relaxed problem by restricting the agent’s off path strategies to have only one deviation - that is the agent can only deviate once but from then on will choose to implement projects if and only if they are good projects. Since the bonus contracts are such that they act as incentives to all deviations, it has to be true that they prevent the agent from such deviations. We then show that if the only off path strategies available to the agent are these deviations, then the incentive compatibility condition for the bad projects have to hold with equality, otherwise the principal can change bonuses to increase profit. Based on that, we obtain a difference equation linking X_{sk} and X_{s+1k} as well as a boundary solution for X_{k-1k}. This gives us a solution for X_{sk} as stated in the proposition. We finally show that the X_{sk} we found by restricting the agent's off-path strategy to one deviations are enough to deter the agent from more complex off-path strategies involving multiple deviations.

Principal's problem

The principal's expected profit under an incentive compatible contract that has the agent implementing project if and only if it is a good project is given by

$$\Pi_k \quad = \quad \theta[\alpha_0\gamma(R - X_{0k}) - c] + \sum_{s=1}^{k-1} \theta^{s+1}\delta^s\{\prod_{m=0}^{s-1}(1-\alpha_m\gamma)\}\{\alpha_s\gamma(R - X_{sk}) - c\}$$

We see in the above expression that the each of the X_{sk} enter negatively in the principal's profit - hence if the principal can reduce any X_{sk} without violating the limited liability or any of the incentive compatibility constraints she would do so.

. The principal's problem is to chose $(X_{sk})_{s=0,1...,k-1}$ and $(V_k(m,s))_{m=0,1..s;s=0,1..k-1}$ to maximize profit subject to the incentive compatibility conditions and the limited liability. This is equivalent to the following cost minimization problem:

$$\min_{(X_{sk})_{s=0}^{k-1},(V_k(m,s))_{m=0,1..s;s=0,1..k-1}} \theta\alpha_0\gamma X_{0,k} + \sum_{s=1}^{k-1} \theta^{s+1}\delta^s(\prod_{m=0}^{s-1}(1-\alpha_m\gamma)(\alpha_s\gamma X_{sk})$$

subject to incentive compatibility for good projects (IC-G)

$$b + \alpha_m \gamma V_k(m, s) + (1 - \alpha_m \gamma)\delta V_k(m + 1, s + 1) \quad \geq \quad \delta V_k(m, s)$$

incentive compatibility for bad projects (IC-B)

$$\delta V_k(m, s) \quad \geq \quad b + \delta V_k(m, s + 1)$$

and limited liability (LL)

$$X_{sk} \quad \geq \quad 0$$

and $V_k(m, s)$ is defined by:

$$\begin{aligned} V_k(m, s) \quad = \max_{1_{Gms}, 1_{BGms}, 1_{Bms}, 1_{Gms} + 1_{BGms} \leq 1} \quad & \{p[1_{Gms}(b + \alpha_m \gamma X_{sk} \\ & + (1 - \alpha_m \gamma)\delta V_k(m + 1, s + 1)) \\ & + 1_{BGms}(1 - 1_{Gms})(b + \delta V_k(m, s + 1)) \\ & + (1 - 1_{BGms})(1 - 1_{Gms})\delta V_k(m, s)] \\ & + (1 - p)[1_{Bms}(b + \delta V_k(m, s + 1)) \\ & + (1 - 1_{Bms})\delta V_k(m, s)]\} \end{aligned}$$

where 1_{Gms} is an indicator function which takes value $= 1$ if the agent selects the good project if it is available and 0 otherwise. Similarly 1_{BGms} stands for the indicator function for the agent's choice regarding bad projects if a good project is available while 1_{Bms} stands for the indicator function for the agent's choice regarding bad projects if a good project is not available.

Suppose the number of public failures is s out of which m failures were in good projects. We define $V_k^T(m,s)$ as the expected profit of the agent if he implements the project if and only if it is a good project from then on.

Then $V_k^T(m,m)$ satisfies the following recurrence relation:

$$\begin{aligned}
V_k^T(m,m) &= p(b+\alpha_m\gamma X_{mk}+(1-\alpha_m\gamma)\delta V_k^T(m+1,m+1))+(1-p)\delta V_k^T(m,m) \\
&= \theta(b+\alpha_m\gamma X_{mk}+(1-\alpha_m\gamma)\delta V_k^T(m+1,m+1)) \\
&= \theta(b+\alpha_m\gamma X_{mk})+\sum_{y=m+1}^{k-1}\theta^{y+1-m}\delta^{y-m}[\prod_{n=m}^{y-1}(1-\alpha_n\gamma)][b+\alpha_y\gamma X_{yk}]
\end{aligned}$$

We can similarly get an expression for $V^T(m,m+1)$ which is given by

$$\begin{aligned}
V_k^T(m,m+1) &= p(b+\alpha_m\gamma X_{m+1k}+(1-\alpha_m\gamma)\delta V_k^T(m+1,m+2))+(1-p)\delta V_k^T(m,m+1) \\
&= \theta(b+\alpha_m\gamma X_{m+1k}+(1-\alpha_m\gamma)\delta V_k^T(m+1,m+2)) \\
&= \theta(b+\alpha_m\gamma X_{m+1k})+\sum_{y=m+1}^{k-2}\theta^{y+1-m}\delta^{y-m}[\prod_{n=m}^{y-1}(1-\alpha_n\gamma)][b+\alpha_y\gamma X_{y+1k}]
\end{aligned}$$

Restriction to one-period deviations:

We start out by restricting the agent to one-period deviations. That is only once will he deviate from the principal's prescribed strategy and from then on he will select the to implement the project if and only if it is a good project. Since the agent is restricted to one-period deviations, the incentive compatibility constraints are that for each of $s=0,1...k-1$ the following inequalities need to hold true.

$$b + \alpha_s \gamma X_{sk} + (1 - \alpha_s \gamma) \delta V_k^T(s+1, s+1) \quad \geq \quad \delta V_k^T(s, s) \qquad \text{(IC-G-O-s)}$$

$$b + \delta V_k^T(s, s+1) \quad \leq \quad \delta V_k^T(s, s) \qquad \text{(IC-B-O-s)}$$

The top inequality (IC-G-O-s) says that the agent prefers to implement a good project if a good project is available. The bottom inequality (henceforth referred to IC-B-O-s) says the payoff from not implementing a project is greater than implementing a bad project.

IC-G-O-s is always satisfied

We first note that the incentive compatibility condition for the good project is always satisfied. To see this, we observe that

$$\begin{aligned}\delta V_k^T(s, s) &= \delta\theta(b + \alpha_s \gamma X_{sk} + (1 - \alpha_s \gamma) \delta V_k^T(s+1, s+1)) \\ &< (b + \alpha_s \gamma X_{sk} + (1 - \alpha_s \gamma) \delta V_k^T(s+1, s+1))\end{aligned}$$

since $0 < \delta, \theta < 1$.

$X_{sk} > 0$ **for all** s

We next observe that $X_{sk} > 0$ for all s. To show this we use a induction argument. That is, we start by showing that this is true for $X_{k-1k} > 0$ and $X_{k-2k} > 0$ and then show that if

$X_{m+1k} > 0$, then $X_{mk} > 0$.

The incentive compatibility condition for the bad project when beliefs are α_{k-1} is given by

$$\delta V_k^T(k-1, k-1) \geq b$$

However we note that

$$V_k^T(k-1, k-1) = \theta(b + \alpha_{k-1}\gamma X_{k-1k})$$

Hence we get

$$X_{k-1k} \geq \frac{b(1-\delta)}{\delta \alpha_{k-1}\gamma p} > 0.$$

Consider $s = k - 2$. The incentive compatibility condition for the bad project when beliefs are α_{k-1} is given by

$$\begin{aligned} \delta V_k^T(k-2, k-2) &\geq b + \delta V_k^T(k-2, k-1) \\ \Rightarrow V_k^T(k-2, k-2) - V_k^T(k-2, k-1) &\geq \frac{b}{\delta} \end{aligned}$$

Using the expressions for $V^T(m, m)$ and $V^T(m, m+1)$, the LHS can be simplified to give

$$\begin{aligned} V_k^T(k-2, k-2) - V_k^T(k-2, k-1) &= \theta\{\alpha_{k-2}\gamma(X_{k-2k} - X_{k-1k})\} \\ &\quad + \theta^2\delta(1 - \alpha_{k-2}\gamma)\{b + \alpha_{k-1}\gamma X_{k-1k}\} \end{aligned}$$

This allows us to obtain

$$\begin{aligned}
\theta\alpha_{k-2}\gamma X_{k-2k} &\geq [\frac{b}{\delta} - \theta^2\delta(1-\alpha_{k-2}\gamma)b] + \theta\alpha_{k-2}\gamma X_{k-1k} - \theta^2\delta(1-\alpha_{k-2}\gamma)\alpha_{k-1}\gamma X_{k-1k} \\
&= b[\frac{1}{\delta} - \theta^2\delta(1-\alpha_{k-2}\gamma)] + \theta\alpha_{k-2}\gamma X_{k-1k}[1-\theta\delta(1-\gamma)] \\
&> 0
\end{aligned}$$

where the second line follows from using Bayes' rule on α_{k-2}. The third line follows from observing that each of $b > 0$, $\frac{1}{\delta} - \theta^2\delta(1-\alpha_{k-2}\gamma) > 0$ and $\theta\alpha_{k-2}\gamma X_{k-1k}[1-\theta\delta(1-\gamma)] > 0$.

General induction step: Assume that each of $X_{k-1k}, X_{k-2k}...X_{m+1k} > 0$. We now show that this implies $X_{mk} > 0$. The incentive compatibility condition for bad projects when beliefs are α_s is given by

$$\delta V_k^T(s,s) \geq b + \delta V_k^T(s,s+1)$$

We can follow similar steps as above and show that

$$\begin{aligned}
V_k^T(s,s) - V_k^T(s,s+1) &= \theta\alpha_s\gamma X_{sk} + X_{s+1k}[\theta^2\delta(1-\alpha_s\gamma)\alpha_{s+1}\gamma - \theta\alpha_s\gamma] + \\
&\quad + \sum_{m=s+2}^{k-1} A_m X_{mk} + \\
&\quad \theta^{k-s}\delta^{k-s-1}(1-\alpha_s\gamma)(1-\alpha_{s+1}\gamma)...(1-\alpha_{k-2}\gamma)\alpha_{k-1}\gamma b
\end{aligned}$$

Here A_m is the coefficient for X_m and is given by

$$\begin{aligned} A_m &= \theta^{m-s+1}\delta^{m-s}\{(1-\alpha_s\gamma)(1-\alpha_{s+1}\gamma)...(1-\alpha_{m-1}\gamma)\alpha_m\gamma \\ &\quad -\theta^{m-s}\delta^{m-s-1}\{(1-\alpha_s\gamma)(1-\alpha_{s+1}\gamma)...(1-\alpha_{m-2}\gamma)\alpha_{m-1}\gamma \\ &= \theta^{m-s+1}\delta^{m-s}\{(1-\alpha_s\gamma)(1-\alpha_{s+1}\gamma)...(1-\alpha_{m-2}\gamma)\alpha_{m-1}(1-\gamma)\gamma \\ &\quad -\theta^{m-s}\delta^{m-s-1}\{(1-\alpha_s\gamma)(1-\alpha_{s+1}\gamma)...(1-\alpha_{m-2}\gamma)\alpha_{m-1}\gamma \\ &= \theta^{m-s}\delta^{m-s-1}\{(1-\alpha_s\gamma)(1-\alpha_{s+1}\gamma)...(1-\alpha_{m-2}\gamma)\alpha_{m-1}\gamma(\theta\delta(1-\gamma)-1) \\ &< 0 \end{aligned}$$

Thus we get that

$$\begin{aligned} \theta\alpha_s\gamma X_{sk} &\geq b[\frac{1}{\delta} - \theta^{k-s}\delta^{k-s-1}(1-\alpha_s\gamma)(1-\alpha_{s+1}\gamma)...(1-\alpha_{k-2}\gamma)\alpha_{k-1}\gamma b] \\ &\quad +X_{s+1k}\theta\alpha_s\gamma[1-\theta\delta(1-\gamma)] + \sum_{m=s+2}^{k-1}(-A_m)X_{mk} \\ &> 0 \end{aligned}$$

where the last equality follows from the observation that $b > 0$, $X_{m+1k}, ...X_{k-1k} > 0$ (from the induction step) as well as the coefficients on $b, X_{s+1k}....X_{k-1k}$ are all positive. Hence we get that $X_{sk} > 0$.

All IC-B-O-s hold with equality

We now argue that all IC-B-OS need to hold with equality.

The argument is by contradiction. Let s be the first instance whereby the inequality is strict that is,

$$\delta V_k^T(s,s) \quad > \quad b + \delta V_k^T(s,s+1)$$

and

$$\delta V_k^T(m,m) \quad = \quad b + \delta V_k^T(m,m+1)$$

for all $0 \leq m < s$.

Rewrite using the definition of $V_k^T(m,m)$ and $V_k^T(m,m+1)$

$$\delta V_k^T(s,s) \quad > \quad b + \delta V_k^T(s,s+1)$$

as

$$\theta(b + \alpha_s \gamma X_{sk} + (1-\alpha_s\gamma)\delta V_k^T(s+1,s+1)) \quad > \quad b + \delta V_k^T(s,s+1)$$

We observe that neither $V_k^T(s+1,s+1)$ nor $V_k^T(s,s+1)$ depend on X_{sk}. Hence it is possible to reduce X_{sk} by a small amount and still have the inequality holding. Since the principal's profit is decreasing in X_{sk}, such an adjustment increases the principal's profit and hence it contradicts X_{sk} being a part of the optimal bonus structure.

It remains to argue that none of the other constraints are violated as a result of this change in X_{sk}. we observe that the expressions for $V_k^T(m,m)$ as well as $V_k^T(m,m+1)$ are not dependent on X_{sk} where $m > s$. Hence changing X_{sk} has no impact on any of the inequalities for $s+1, s+2 ... k-1$.

What about the incentive constraints for $m < s$? We know that for all such m the following relation holds.

$$\delta V_k^T(m,m) = b + \delta V_k^T(m,m+1)$$

Reducing X_s by ϵ decreases $\delta V_k^T(m,m)$ by $\epsilon\{\theta^{s-m}\delta^{s-m-1}(1-\alpha_m\gamma)...(1-\alpha_{s-1}\gamma)\alpha_s\gamma$ while decreases $\delta V_k^T(m,m+1)$ by $\epsilon\{\theta^{s-m-1}\delta^{s-m-2}(1-\alpha_m\gamma)...(1-\alpha_{s-2}\gamma)\alpha_{s-1}\gamma$. Observe that

$$(1-\alpha_{s-1}\gamma)\alpha_s = (1-\gamma)\alpha_{s-1}$$

and hence

$$(1-\alpha_m\gamma)...(1-\alpha_{s-1}\gamma)\alpha_s\gamma = (1-\alpha_m\gamma)...(1-\alpha_{s-2}\gamma)\alpha_{s-1}\gamma(1-\gamma).$$

Thus the fall in $\delta V_k^T(m,m)$ is smaller than the $\delta V_k^T(m,m+1)$ and hence the incentive compatibility constraint for bad project continues to hold.

Recurrence relation:

We have shown that all the IC-B–O-s need to hold with equality. We now prove the following recurrence relation:

$$X_{sk} = \frac{b(1-\delta)}{\delta\gamma\alpha_s p} + X_{s+1k} + b$$

along with the boundary condition:

$$X_{k-1k} = \frac{b(1-\delta)}{\delta\gamma\alpha_{k-1} p}$$

The IC-B-O-$k-1$ gives us

$$\begin{aligned}
\delta V_k^T(k-1,k-1) &= b \\
\Rightarrow \theta\delta(b+\alpha_{k-1}\gamma X_{k-1,k}) &= b \\
\Rightarrow p\delta(b+\alpha_{k-1}\gamma X_{k-1,k}) &= b(1-\delta(1-p)
\end{aligned}$$

Simplifying we get,

$$X_{k-1k} = \frac{b(1-\delta)}{\delta\gamma\alpha_{k-1}p}.$$

To prove the recurrence relation we use induction on s.

For $s=k-2$, the IC-B-OS gives us

$$\delta V_k^T(k-2,k-2) = b+\delta V_k^T(k-2,k-1)$$

This can be rewritten as

$$\begin{aligned}
\delta p(b+\alpha_{k-2}\gamma X_{k-2}+(1-\alpha_{k-2}\gamma)\delta V_k^T(k-1,k-1)) &= b(1-\delta+\delta p)+ \\
&\quad \delta V_k^T(k-2,k-1)((1-\delta+\delta p)
\end{aligned}$$

To simplify the above expression, we observe

$$V_k^T(k-2,k-1)(1-\delta+\delta p) = p(b+\alpha_{k-2}\gamma X_{k-1k})$$

and the IC-B-O-$k-1$ gives us

$$\delta V_k^T(k-1,k-1) = b$$

Thus we get,

$$\delta p b + \delta p \alpha_{k-2} \gamma X_{k-2k} = b(1-\delta+\delta p) + \delta p(b + \alpha_{k-1}\gamma X_{k-1k})$$
$$-\delta p(1-\alpha_{k-2}\gamma) b$$

which gives us

$$X_{k-2k} = \frac{b(1-\delta)}{\delta p \gamma \alpha_{k-2}} + X_{k-1k} + b$$

which verifies the recurrence equation above for $s = k-2$.

We now assume that the recurrence relation holds for $s+1, s+2..., k-2, k-1$ and show that it holds for X_{sk} as well.

The IC-B-O-s gives us

$$\delta V_k^T(s,s) = b + \delta V_k^T(s, s+1)$$

We observe that

$$V_k^T(s,s) = \theta(b + \alpha_s \gamma X_{sk} + (1-\alpha_s\gamma)\delta V_k^T(s+1, s+1))$$

Hence

$$\delta\theta(b + \alpha_s \gamma X_{sk} + (1-\alpha_s\gamma)\delta V_k^T(s+1, s+1)) = b + \delta V_k^T(s, s+1)$$

Multiplying throughout by $1-\delta+\delta p$ and simplifying we get,

$$\begin{aligned}\delta p \alpha_s \gamma X_{sk} &= b(1-\delta) + (1-\delta+\delta p)\delta V_k^T(s, s+1) \\ &\quad -\delta^2 p(1-\alpha_s\gamma)V_k^T(s+1, s+1)\end{aligned}$$

We see that

$$\begin{aligned}(1-\delta+\delta p)V_k^T(s, s+1) &= p[b + \alpha_s \gamma X_{s+1k} \\ &\quad +p(1-\alpha_s\gamma)\delta V_k^T(s+1, s+2)]\end{aligned}$$

Inserting this in the above equation we get,

$$\begin{aligned}\delta p \alpha_s \gamma X_{sk} &= b(1-\delta) + \delta[pb + p\alpha_s \gamma X_{s+1k} + \\ &\quad +p(1-\alpha_s\gamma)(\delta V_k^T(s+1, s+2) - \delta V_k^T(s+1, s+1))]\end{aligned}$$

We know that

$$\delta V_k^T(s+1, s+1) = b + \delta V_k^T(s+1, s+2)$$

This gives us

$$\begin{aligned}\delta p \alpha_s \gamma X_{sk} &= b(1-\delta) + \delta[pb + p\alpha_s \gamma X_{s+1k} \\ &\quad -p(1-\alpha_s\gamma)b] \\ &= b(1-\delta) + \delta p \alpha_s \gamma [X_{s+1k} + b]\end{aligned}$$

which gives us

$$X_{sk} = \frac{b(1-\delta)}{\delta p \alpha_s \gamma} + X_{s+1k} + b$$

which proves the recurrence relation.

Deriving the formula stated in the proposition

We thus see that

$$\begin{aligned} X_{sk} &= \frac{b(1-\delta)}{\delta p \alpha_s \gamma} + X_{s+1k} + b \\ &= \frac{b(1-\delta)}{\delta p \alpha_s \gamma} + \frac{b(1-\delta)}{\delta p \alpha_{s+1} \gamma} + X_{s+2k} + b + b \\ &= (k-1-s)b + \sum_{m=s}^{k-1} \frac{b(1-\delta)}{\delta p \gamma \alpha_m}. \end{aligned}$$

Showing this is sufficient to deter more complex off path strategies for the agent

We now verify that $(X_{sk})_{s=0,1..k-1}$ we found above is sufficient to guarantee that the agent won't want to deviate from the prescribed strategy even if he had access to more complex strategies than one deviations.

The idea is to use induction to show that $(X_{sk})_{s=0,1..k-1}$ is enough to prevent the agent from taking up bad projects regardless of the beliefs of the agent and the principal - that is we show that

$$\delta V_k(m,s) \geq b + \delta V_k(m,s+1).$$

where $m = 0, 1...s$ and $s = 0, 1...k-1$.

Note that it suffices to make sure that the incentive compatibility condition for the bad project holds since in that case, there is no gain to choosing not to implement a project when the project available is good as in the next period the agent's payoff is going to be the same as the previous period but now discounted.

Fix $s = k - 1$. We want to show that for $m = 0, 1 ... k - 1$

$$\delta V_k(m, k-1) \geq b.$$

One possible strategy for the agent is that he selects not to implement a project if the project available is bad and implement the good project if it is available. Since $V_k(m, k-1)$ is the maximum payoff possible, it has to be true that $V_k(m, k-1)$ gives a weakly higher payoff than following the above strategy that is

$$\delta V_k(m, k-1) \geq \delta\theta(b + \alpha_m \gamma X_{k-1k})$$

Since $m \leq k - 1$, we get that $\alpha_m \geq \alpha_{k-1}$ and hence

$$\begin{aligned} \delta V_k(m, k-1) &\geq \delta\theta(b + \alpha_{k-1}\gamma X_{k-1k}) \\ &= \delta\theta(b + \alpha_{k-1}\gamma \frac{b(1-\delta)}{\delta\alpha_{k-1}\gamma p}) \\ &= \delta\theta b(1 + \frac{(1-\delta)}{\delta p}) \\ &= \delta\theta b \frac{1-\delta+\delta p}{\delta p} \\ &= b \end{aligned}$$

where the last line follows from noting $\theta \equiv \frac{p}{1-\delta(1-p)}$.

Fix $s = k - 2$. We want to show that for $m = 0, 1...k - 2$

$$\delta V_k(m, k-2) \quad \geq \quad b + \delta V_k(m, k-1).$$

To reduce notation, we are going to refer to $V_k(m, k-2) \equiv V_{m,k-2}$ and so on for the remaining part of this proof. One possible strategy for the agent is that he selects the safe project if the risky project is bad and the risky project if it is a good project. Since $V_{m,k-2}$ is the maximum payoff possible, it has to be true that $V_{m,k-2}$ gives a weakly higher payoff than following the above strategy that is

$$V_{m,k-2} \quad \geq \quad \theta(b + \alpha_m \gamma X_{k-2k} + (1 - \alpha_m \gamma)\delta V_{m+1,k-1}).$$

Hence it is enough to show that

$$\delta(b + \alpha_m \gamma X_{k-2k} + (1 - \alpha_m \gamma)\delta V_{m+1,k-1}) \quad \geq \quad \frac{1}{\theta}(b + \delta V_{m,k-1})$$

Simplifying the expression we get,

$$\delta \alpha_m \gamma X_{k-2k} + (1 - \alpha_m \gamma)\delta^2 V_{m+1,k-1} \quad \geq \quad \frac{b}{\theta} - \delta b + \frac{\delta}{\theta} V_{m,k-1}$$

We know that

$$X_{k-2k} \quad = \quad \frac{b(1-\delta)}{\delta p \gamma \alpha_{k-2}} + X_{k-1k} + b$$

and

$$V_{m,k-1} \quad = \quad \theta(b + \alpha_m \gamma X_{k-1k})$$

Using the above two equalities to simplify the previous inequality

$$\delta\alpha_m\gamma X_{k-2k} + (1-\alpha_m\gamma)\delta^2 V_{m+1,k-1} \quad \geq \quad \frac{b}{\theta} - \delta b + \frac{\delta}{\theta} V_{m,k-1}$$

which is the same as

$$\frac{\alpha_m}{\alpha_{k-2}} \frac{b(1-\delta)}{p} + \delta\alpha_m\gamma b + (1-\alpha_m\gamma)\delta^2 V_{m+1,k-1} \quad \geq \quad \frac{b}{\theta}$$

which can be further simplified to yield

$$\frac{b(1-\delta)}{p}[\frac{\alpha_m}{\alpha_{k-2}} - 1] - \delta b(1-\alpha_m\gamma) + (1-\alpha_m\gamma)\delta^2 V_{m+1,k-1} \quad \geq \quad 0$$

which gives us

$$\frac{b(1-\delta)}{p}[\frac{\alpha_m}{\alpha_{k-2}} - 1] + \delta(1-\alpha_m\gamma)[\delta V_{m+1,k-1} - b] \quad \geq \quad 0$$

But $m \leq k-2$ which gives us $\alpha_m \geq \alpha_{k-2}$ and we also get from the previous step that $\delta V_{m+1,k-1} - b \geq 0$ which verifies that

$$\frac{b(1-\delta)}{p}[\frac{\alpha_m}{\alpha_{k-2}} - 1] + \delta(1-\alpha_m\gamma)[\delta V_{m+1,k-1} - b] \quad \geq \quad 0$$

and hence

$$\delta V(m, k-2) \quad \geq \quad b + \delta V(m, k-1).$$

We now want to show that if for all $m = 0, 1...r$ and $r = s+1, s+2...k-1$

$$\delta V(m,r) \geq b + \delta V(m, r+1).$$

then the following relation holds for all $m = 0, 1....s$:

$$\delta V(m,s) \geq b + \delta V(m, s+1).$$

We proceed similarly as before. We know that

$$V_{m,s} \geq \theta(b + \alpha_m \gamma X_s + (1-\alpha_m\gamma)\delta V_{m+1,s+1}).$$

Hence it is enough to show that

$$\delta(b + \alpha_m \gamma X_s + (1-\alpha_m\gamma)\delta V_{m+1,s+1}) \geq \frac{1}{\theta}(b + \delta V_{m,k-1})$$

which is the same as showing

$$\delta\alpha_m \gamma X_{sk} + (1-\alpha_m\gamma)\delta^2 V_{m+1,s+1} \geq \frac{b}{\theta} - \delta b + \frac{\delta}{\theta} V_{m,s+1}$$

We can use the induction assumption to get

$$V_{m,s+1} = \theta(b + \alpha_m \gamma X_{sk} + (1-\alpha_m\gamma) V_{m+1,s+2})$$

and also

$$X_{sk} = X_{s+1k} + \frac{b(1-\delta)}{\delta\alpha_s\gamma p} + b$$

to simplify the above inequality as

$$\frac{b(1-\delta)}{p}[\frac{\alpha_m}{\alpha_s}-1]+\delta(1-\alpha_m\gamma)[\delta V_{m+1,s+1}-b-\delta V_{m+1,s+2}] \quad \geq \quad 0$$

We get $\frac{\alpha_m}{\alpha_s}-1\geq 0$ since $m\leq s$ and also $\delta V_{m+1,s+1}-b-\delta V_{m+1,s+2}>0$ from the induction assumption. Hence we have verified that indeed

$$\frac{b(1-\delta)}{p}[\frac{\alpha_m}{\alpha_s}-1]+\delta(1-\alpha_m\gamma)[\delta V_{m+1,s+1}-b-\delta V_{m+1,s+2}] \quad \geq \quad 0$$

and this concludes the induction argument.

Proof of Lemma 1

The expression for MB_k^{SB} is given by

$$MB_k^{SB} \quad = \quad \theta^{k+1}\delta^k[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma(R-X_{kk+1})-c]$$

The lemma follows from observing that each of the terms above are decreasing in k. We note that δ as well as θ lie between 0 and 1. Hence θ^{k+1} and δ^k are both decreasing in k.

Second, since $(1-\alpha_i\gamma)$ lies between 0 and 1, the product

$$\prod_{m=0}^{k-1}(1-\alpha_m\gamma)$$

also lies in between 0 and 1 and hence increasing k multiplies this with a term which is between 0 and 1 and thus reduces it further.

From Bayes' rule we get,

$$\alpha_k \quad = \quad \frac{(1-\gamma)^k\alpha_0}{(1-\gamma)^k\alpha_0+(1-\alpha_0)}.$$

and thus α_k is a decreasing function of k.

Finally

$$-X_{kk+1} \;=\; -\frac{b(1-\delta)}{\delta\gamma\alpha_k p}$$

is also decreasing in k since α_k is decreasing in k.

Proof of Lemma 2

The expression for "marginal cost" is given as

$$MC_k^{SB} \;=\; \sum_{s=1}^{k}\theta^s\delta^{s-1}[\alpha_0(1-\gamma)^{s-1}\gamma(b+\frac{b(1-\delta)}{\delta\alpha_k\gamma p})]$$

We observe that

$$\theta^s\delta^{s-1}[\alpha_0(1-\gamma)^{s-1}\gamma(b+\frac{b(1-\delta)}{\delta\alpha_k\gamma p})]$$

is positive and is also increasing in k since α_k is decreasing in k. Hence increasing k leads to an increase in the marginal cost - first, each of the terms above increase due to α_k being a decreasing function of k and second, a positive term gets added since we are summing from 1 to k.

Proof of Proposition 2

We see that

$$\begin{aligned}\triangle\Pi_k \;&=\; \theta^{k+1}\delta^k[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma(R-X_{k,k+1})-c]\\ &\quad+\sum_{s=1}^{k}\theta^s\delta^{s-1}\alpha_0(1-\gamma)^{s-1}\gamma(X_{s-1,k}-X_{s-1,k+1})\end{aligned}$$

Using the definitions of marginal benefit and marginal cost as defined in the text, we can see that this can be written as

$$\triangle\Pi_k \equiv MB_k^{SB} - MC_k^{SB}$$

Lemma 1 says that MB_k^{SB} is decreasing in k while lemma 2 says that MC_k^{SB} is increasing in k. Thus we get that $\triangle\Pi_k$ is decreasing in k.

As we increase k, $\alpha_k\gamma(R - X_{kk+1}) - c$ becomes negative for some finite k which implies that the "marginal benefit" becomes negative for some finite k. The "marginal cost" on the other hand is always positive and is strictly increasing. Assumption 2 guaranteed that $MB_0^{SB} > 0 = MC_0^{SB}$ which suggested that some experimentation is optimal in the second best. As we increase k, there exists a value of k, say k^* for which $MB_{k^*}^{SB} \geq MC_{k^*}^{SB}$ and $MB_{k^*+1}^{SB} < MC_{k^*+1}^{SB}$. The optimal number of trials is given by k^*. To see this, note that if $k > k^*$, the principal can increase expected payoff by reducing k since at such a k, $MB_k^{SB} < MC_k^{SB}$. However if $k < k^*$, then $MB_k^{SB} > MC_k^{SB}$ and hence the principal can increase expected payoff by increasing k.

Proof of Proposition 3

In the complete information benchmark, there are no bonuses paid. Hence $MC_k^{CI} = 0$ for all k while the marginal benefit is given by

$$MB_k^{CI} = \theta^{k+1}\delta^k[\prod_{m=0}^{k-1}(1 - \alpha_m\gamma)][\alpha_k\gamma R - c]$$

Since $X_{kk+1} > 0$ we see that $MB_k^{CI} > MB_k^{SB}$. Thus in the complete information benchmark, both "marginal benefit" is higher and the "marginal cost" is lower compared to the second best. Hence the optimal of trials will be higher as well. Note that even if

$MB_k^{CI} > MB_k^{SB}$ it is still true that MB_k^{CI} is decreasing in k - the argument is similar to that presented in the proof of Lemma 2 - and hence experimentation is terminated after a finite number of failures even in the complete information benchmark.

Proof for the Comparative Statics

Comparative statics with respect to α_0

Lemma A.5.1: MB_k^{SB} is increasing in α_0 for all k for which $MB_k^{SB} > 0$.

Proof: We see that

$$MB_k^{SB} \equiv \theta^{k+1}\delta^k[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma(R-X_{kk+1})-c]$$

We examine separately the terms which are a function of α_0:

$$[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma(R-X_{kk+1})-c]$$

This can be simplified as

$$[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)]\alpha_k\gamma(R-X_{k,k+1})-[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)]c$$

We note that

$$\prod_{m=0}^{k-1}(1-\alpha_m\gamma)\alpha_k\gamma = \alpha_0(1-\gamma)^k\gamma$$

and hence is increasing in α_0.

From equation (1), we see that α_k is also increasing in α_0.

Finally

$$-X_{kk+1} = -\frac{b(1-\delta)}{\delta\gamma\alpha_k p}$$

is increasing in α_0 since α_k is increasing in α_0. Thus $[\prod_{m=0}^{k-1}(1-\alpha_m\gamma)][\alpha_k\gamma(R-X_{kk+1})]$ is increasing in α_0.

Next we observe that

$$\prod_{m=0}^{k-1}(1-\alpha_m\gamma) = 1-\alpha_0+\alpha_0(1-\gamma)^k$$

Taking derivative of this expression with respect to α_0, we get $-1+(1-\gamma)^k<0$ - hence $\prod_{m=0}^{k-1}(1-\alpha_m\gamma)$ is decreasing in $\alpha_0$which implies that $-\prod_{m=0}^{k-1}(1-\alpha_m\gamma)c$ is increasing in α_0. Thus both of the components in the expression for MB_k^{SB} is increasing in α_0 which gives us the result.

Lemma A.5.2: Fix $k \geq 1$. An increase in α_0 can lead to a increase in MC_k^{SB}.

Proof: We start by noting

$$\begin{aligned} MC_k^{SB} &= \sum_{s=1}^{k}\theta^s\delta^{s-1}\alpha_0(1-\gamma)^{s-1}\gamma(b+\frac{b(1-\delta)}{\delta\alpha_k\gamma p}) \\ &= \alpha_0(1+\frac{(1-\delta)}{\delta\alpha_k\gamma p})[\sum_{s=1}^{k}\theta^s\delta^{s-1}b(1-\gamma)^{s-1}\gamma] \end{aligned}$$

The portion that is dependent on α_0 is given by $\alpha_0(1+\frac{(1-\delta)}{\delta\alpha_k\gamma p})$. The derivative of this expression with respect to α_0 is given by

$$1+\{\frac{1-\delta}{\delta}.\frac{1}{\gamma p}\}\{1-\frac{1}{(1-\gamma)^{k-1}}\}$$

which can be positive - hence an increase in α_0 can lead to a increase in MC_k^{SB}.

Alternatively observe that for $\delta = 1$, MC_k^{SB} simplifies to $\sum_{s=1}^{k} \alpha_0(1-\gamma)^{s-1}\gamma b$ which is an increasing function of α_0.

Lemma A.5.3 : An increase in α_0 leads to an increase in the expected payoff for the principal.

The principal's expected profit for k trials is given by

$$\Pi_k = \theta[\alpha_0\gamma(R - X_{0k}) - c] + \sum_{s=1}^{k-1} \theta^{s+1}\delta^s\{\prod_{m=0}^{s-1}(1-\alpha_m\gamma)\}\{\alpha_s\gamma(R - X_{sk}) - c\}$$

Fix k. Then an increase in α_0 leads to an increase in the Π_k. The proof is similar to showing that the "marginal benefit" is an increasing function of α_0(Lemma A.5.2). The only difference is we have X_{sk} where $s = 0, 1...k-1$ in place of X_{kk+1}. However if we hold fixed k, then X_{sk} is a decreasing function of α_0 just as X_{kk+1} is decreasing function of α_0 and hence analogous arguments hold.

Lemma A.5.4 : An increase in α_0 leads to an increase in the number of trials in the second best.

Define $k^*(\alpha_0)+1$ as the optimal number of trials in the second best when initial prior about the agent being of high ability is given by α_0.

We have $\Pi_{k^*(\alpha_0)+1} - \Pi_{k^*(\alpha_0)} \equiv \triangle\Pi_{k^*(\alpha_0)} \geq 0$, since $k^*(\alpha_0) + 1$ is the optimal number of trials when prior is given by α_0. Using the expressions for $MB_{k^*(\alpha_0)}^{SB}$ and $MC_{k^*(\alpha_0)}^{SB}$, we can rewrite this condition as

$$\theta^{k^*(\alpha_0)+1}\delta^{k^*(\alpha_0)}(1-\alpha_0\gamma)..(1-\alpha_{k^*(\alpha_0)-1}\gamma)[\alpha_{k^*(\alpha_0)}\gamma(R-X_{k^*(\alpha_0)k^*(\alpha_0)+1})-c]$$
$$+\sum_{s=1}^{k^*(\alpha_0)}\theta^s\delta^{s-1}\alpha_0(1-\gamma)^{s-1}\gamma(X_{s-1\,k^*(\alpha_0)}-X_{s-1\,k^*(\alpha_0)+1}) \;\; \geq \;\; 0$$

Since $X_{kk+1}=\frac{b(1-\delta)}{\delta\alpha_k\gamma p}$ and $X_{s-1\,k}-X_{s-1\,k+1}=-b-\frac{b(1-\delta)}{\delta\alpha_k\gamma p}$, the above expression can be rewritten as

$$\theta^{k^*(\alpha_0)+1}\delta^{k^*(\alpha_0)}\alpha_0(1-\gamma)^{k^*(\alpha_0)}\gamma R-(1-\alpha_0+\alpha_0(1-\gamma)^k)(\frac{b(1-\delta)}{\delta p}+c)$$
$$-\sum_{s=1}^{k^*(\alpha_0)}\theta^s\delta^{s-1}\alpha_0(1-\gamma)^{s-1}\gamma(b+\frac{b(1-\delta)}{\delta\alpha_{k^*(\alpha_0)}\gamma p}) \;\; \geq \;\; 0$$

from which we obtain

$$\theta^{k^*(\alpha_0)+1}\delta^{k^*(\alpha_0)}(1-\gamma)^{k^*(\alpha_0)}R-\sum_{s=1}^{k^*(\alpha_0)}\theta^s\delta^{s-1}(1-\gamma)^{s-1}b \;\; > \;\; 0.$$

We can rewrite $\triangle\Pi_{k^*(\alpha_0)}$ as

$$\begin{aligned}\triangle\Pi_{k^*(\alpha_0)} \;=\; & \theta^{k^*(\alpha_0)+1}\delta^{k^*(\alpha_0)}\alpha_0(1-\gamma)^{k^*(\alpha_0)}\gamma R \\ & -(1-\alpha_0+\alpha_0(1-\gamma)^{k^*(\alpha_0)})(\frac{b(1-\delta)}{\delta p}+c) \\ & -\sum_{s=1}^{k^*(\alpha_0)}\theta^s\delta^{s-1}\alpha_0(1-\gamma)^{s-1}\gamma b \\ & -\sum_{s=1}^{k^*(\alpha_0)}\theta^s\delta^{s-1}(1-\gamma)^{s-1}\frac{(1-\alpha_0+\alpha_0(1-\gamma)^{k^*(\alpha_0)})}{(1-\gamma)^{k^*(\alpha_0)}}\frac{b(1-\delta)}{\delta p}\end{aligned}$$

Using the envelope theorem, we obtain

$$\begin{aligned}
\frac{\partial \triangle \Pi_{k^*(\alpha_0)}}{\partial \alpha_0} &= \theta^{k^*(\alpha_0)+1}\delta^{k^*(\alpha_0)}(1-\gamma)^{k^*(\alpha_0)}\gamma R \\
&\quad -(-1+(1-\gamma)^{k^*(\alpha_0)})(\frac{b(1-\delta)}{\delta p}+c) \\
&\quad -\sum_{s=1}^{k^*(\alpha_0)} \theta^s \delta^{s-1}(1-\gamma)^{s-1}\gamma b \\
&\quad -\sum_{s=1}^{k^*(\alpha_0)} \theta^s \delta^{s-1}(1-\gamma)^{s-1}\frac{(-1+(1-\gamma)^{k^*(\alpha_0)})}{(1-\gamma)^{k^*(\alpha_0)}}\frac{b(1-\delta)}{\delta p}
\end{aligned}$$

Since $-1+(1-\gamma)^{k^*(\alpha_0)} < 0$ and

$$\theta^{k^*(\alpha_0)+1}\delta^{k^*(\alpha_0)}(1-\gamma)^{k^*(\alpha_0)}R - \sum_{s=1}^{k^*(\alpha_0)} \theta^s \delta^{s-1}(1-\gamma)^{s-1} b \quad > \quad 0.$$

we obtain that

$$\frac{\partial \triangle \Pi_{k^*(\alpha_0)}}{\partial \alpha_0} \quad > \quad 0$$

Hence an increase in α_0 increases $\triangle \Pi_{k^*(\alpha_0)}$. Since $\triangle \Pi_{k^*(\alpha_0)} \geq 0$, this implies that an increase in prior leads to an increase in the number of trials (from proposition 2).

Comparative statics with respect to c

Lemma A.5.5: MB_k^{SB} is decreasing in c and MC_k^{SB} is independent of c. Hence an increase in c leads to a decrease in the number of trials.

The first part of the lemma follows from observing that

$$MC_k^{SB} \;=\; \sum_{s=1}^{k} \theta^s \delta^{s-1} \alpha_0 (1-\gamma)^{s-1} \gamma \Big(b + \frac{b(1-\delta)}{\delta \alpha_k \gamma p}\Big)$$

is independent of c while

$$MB_k^{SB} \;=\; \theta^{k+1} \delta^k \Big[\prod_{m=0}^{k-1} (1-\alpha_m \gamma)\Big](\alpha_k \gamma (R - X_{kk+1}) - c)$$

is clearly a decreasing function of c for a given value of k.

The second part of the lemma is a consequence of proposition 2.

Lemma A.5.6: An increase in c leads to a decrease in the expected payoff of the principal.

Let $c^H > c^L$ and let $k^*(c)$ denote the optimal number of trials when cost of implementing a project is given by c. Let $\Pi_k(c)$ denote the principal's expected payoff from a k-trial contract when the cost of implementing project is c. We observe that

$$\Pi_k(c) \;=\; \theta[\alpha_0 \gamma (R - X_{0k}) - c] + \sum_{s=1}^{k-1} \theta^{s+1} \delta^s \Big\{\prod_{m=0}^{s-1} (1-\alpha_m \gamma\Big\}\{\alpha_s \gamma (R - X_{sk}) - c\}$$

Holding fixed k, we observe that$\Pi_k(c)$ is a decreasing function of c.

From Lemma A.5.6, $k^*(c^L) \geq k^*(c^H)$.

Next observe that $\Pi_{k^*(c^L)}(c^L) \geq \Pi_{k^*(c^H)}(c^L)$, since $k^*(c^L)$ denote the optimal number of trials when cost of implementing a project is given by c^L.

Thus we get $\Pi_{k^*(c^L)}(c^L) \geq \Pi_{k^*(c^H)}(c^L) \geq \Pi_{k^*(c^H)}(c^H)$ which concludes the proof.